The Association
of
Engineering and Shipbuilding Draughtsmen.

Design Examples for Young Draughtsmen.

By J. W. HAMILTON.

Published by The Association of Engineering and Shipbuilding Draughtsmen,
Onslow Hall, Little Green, Richmond, Surrey.

SESSION 1947-48.
(Revised).

Price 3/-

Printed by Milne, Tannahill & Methven, Ltd. (T.U.), 12-14 Mill Street, Perth.

ADVICE TO INTENDING AUTHORS OF A.E.S.D. PRINTED PAMPHLETS.

Pamphlets submitted to the National Technical Sub-Committee for consideration with a view to publication in this series should not exceed 10,000 to 15,000 words and about 20 illustrations. The aim should be the presentation of the subject clearly and concisely, avoiding digressions and redundance. Manuscripts are to be written in the third person. Copies of an article entitled "Hints on the Writing of Technical Articles" can be obtained from the Editor of *The Draughtsman*.

Drawings for illustrations should be done either on a good plain white paper or tracing cloth, deep black Indian ink being used. For ordinary purposes they should be made about one-and-a-half times the intended finished size, and it should be arranged that wherever possible these shall not be greater than a single full page of the pamphlet, as folded pages are objectionable, although, upon occasion, unavoidable. Where drawings are made larger, involving a greater reduction, the lines should be made slightly heavier and the printing rather larger than normal, as the greater reduction tends to make the lines appear faint and the printing excessively small in the reproduction. In the case of charts or curves set out on squared paper, either all the squares should be inked in, or the chart or curve should be retraced and the requisite squares inked in. Figures should be as self-evident as possible. Data should be presented in graphical form. Extensive tabular matter, if unavoidable, should be made into appendices.

Authors of pamphlets are requested to adhere to the standard symbols of the British Standards Institution, where lists of such standard symbols have been issued, as in the case of the electrical and other industries, and also to the *British Standard Engineering Symbols and Abbreviations*, No. 560, published by the B.S.I. in 1934 at 5/-. Attention might also be given to mathematical notation, where alternative methods exist, to ensure the minimum trouble in setting up by the printer.

The value of the pamphlet will be enhanced by stating where further information on the subject can be obtained. This should be given in the form of footnotes or a bibliography, including the name and initials of the author, title, publisher, and year of publication. When periodicals are referred to, volume and page also should be given. References should be checked carefully.

Manuscripts, in the first instance, should be submitted to the Editor *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

For Pamphlets, a grant of £20 will be made to the Author, but special consideration will be given in the case of much larger pamphlets which may involve more than the usual amount of preparation.

The Association
of
Engineering and Shipbuilding Draughtsmen.

Design Examples for Young Draughtsmen.

By J. W. HAMILTON.

Published by the Association of Engineering and Shipbuilding Draughtsmen,
Onslow Hall, Little Green, Richmond, Surrey.

SESSION 1947-48.
(Revised).

6/55.

CRITICAL SPEED

$$n \cong 300\sqrt{\frac{K}{G}} \cong \sqrt{\frac{1}{f}} \text{ RPM.}$$

f = Static deflection under load G. (in)
K = Load req^d to produce unit deflection (lb/in)
G = Modulus. (lb/in²).

CONTENTS

Note.—This fresh edition has been issued owing to continued popular demand. As it has been impossible to trace the author, the National Technical Sub-Committee has confined itself to a very few necessary corrections.—*Editor.*

INTRODUCTION.

This pamphlet has been prepared principally for the younger members of the Association, although the subject matter may also be of some interest to other members.

The following collection of examples is not exhaustive, but it covers the more common features of general engineering design, and, in the author's opinion, should form a sound basis for a more detailed study of the subject.

While the pamphlet is suitable for those studying for the Higher National Certificate in Mechanical Engineering, the drawing office viewpoint has not been neglected.

In the drawing office one occasionally meets the draughtsman who designs "with the eye" or who guesses machine proportions from practical experience. This method, in addition to being dangerous, is unscientific. It is, moreover, an impossible method for the young draughtsman, owing to lack of experience.

Designing on theoretical lines with an eye to practical considerations *after* the theory has been tested is the most rational method. To obtain a thorough grasp of design principles, the most reliable system is by means of the worked example. Although text books are useful, a study of numerous proofs and formulae does not imprint the theory so quickly on the mind as a clear cut application to some familiar machine part.

It is not claimed that the examples in this pamphlet will provide the student with a ready-made knowledge of design, but they should assist materially in focussing his attention on the more salient features.

The examples have been classified, for convenience, into various sections. Where practicable, the appropriate formulae precede each section, and the examples are interspersed with explanatory notes.

No attempt has been made to theorise, and it is assumed that the student has sufficient knowledge of applied mechanics to understand the principles involved. Where such knowledge is incomplete, the student is recommended to study these examples in conjunction with a standard text-book.

The Appendix gives a few tables which should be useful when working out similar problems, particularly the first three tables, which are taken from Unwin's *Machine Design*, and give safe values for working stresses.

The author wishes to acknowledge his indebtedness to Wm. Ferguson, Esq., B.Sc., A.M.I.M.E., mechanical lecturer at Coatbridge Technical College, for permission to use some of the examples.

DESIGN EXAMPLES FOR YOUNG DRAUGHTSMEN.

J. W. Hamilton.

Section I.

SHAFTS AND COUPLINGS.

Formulae.

$$T = \frac{H.P. \times 63{,}000}{N}$$

$$\frac{T}{J} = \frac{f_s}{r} = \frac{G\,\theta}{l}$$

$$Z_t = \frac{T}{f_s}$$

For Solid Circular Sections,

$$T = \frac{\pi}{16}\, d^3 f_s = Z_t f_s$$

For Hollow Circular Sections,

$$T = \frac{\pi}{16} \left(\frac{D^4 - d_1{}^4}{D}\right) f_s$$

For Square Sections,

$$T = \cdot 208\; s^3 f_s$$

For Rectangular Sections,

$$T = 2/9\; Bb^2 f_s \text{ (approx.)}$$

Notation.

T	=	Torque transmitted or twisting moment (in. lbs.).
HP	=	Horse-power transmitted.
N	=	Number of revs. per minute.
J	=	Polar moment of inertia (in.^{4}).
f_s	=	Max. shear stress (lb./in.^{2}).
r	=	Outside radius of shaft (ins.).
G	=	Modulus of rigidity (lbs./in.^{2}).
θ	=	Angle of twist (radian measure).
l	=	Length of shaft (ins.).
Z_t	=	Modulus of section for twisting (in.^{3}).

d = Dia. of solid shaft (ins.).
D = Outside dia. of hollow shaft (ins.).
d_1 = Inside dia. of hollow shaft (ins.).
s = Side of square (ins.).
B = Larger side of rectangle (ins.).
b = Smaller side of rectangle (ins.).

Note.—Values for J and Z_t for circular sections may be taken direct from most pocket books.

Example 1.—(*a*) Find the dia. of a M.S. shaft to transmit 60 H.P. at 150 R.P.M., taking $f_s = 8000$ lb./in.². (*b*) What is the angle of twist over a 10-ft. length? $G = 12{\cdot}5 \times 10^6$ lb./in.²

$$T = \frac{HP \times 63{,}000}{N}$$

$$= \frac{60 \times 63{,}000}{150} = \mathbf{25200\ in.\ lbs.}$$

$$T = \frac{\pi}{16} d^3 f_s$$

$$\therefore\ 25{,}200 = \frac{\pi}{16} d^3 \times 8000$$

$$\therefore\ d^3 = \frac{25{,}200 \times 16}{\pi \times 8000} = 16{\cdot}1 \text{ approx.}$$

$$\therefore\ d = 2{\cdot}52'', \text{ say } \mathbf{2\tfrac{1}{2}''}$$

(*b*)
$$\frac{T}{J} = \frac{G\,\theta}{l}$$

$$J = \frac{\pi d^4}{32} = 3{\cdot}835 \text{ in.}^4 \text{ (from tables).}$$

$$\therefore\ \frac{25{,}200}{3{\cdot}835} = \frac{12{\cdot}5 \times 10^6 \times \theta}{10 \times 12}$$

From this, θ = ·063 radians.
One radian = 57·3°
∴ Angle of twist = $57{\cdot}3 \times {\cdot}063 = 3{\cdot}6°$

Example 2.—A hollow steel shaft 9″ inside dia., 12″ outside dia., transmits 3000 H.P. at 120 R.P.M. What is the maximum stress in the shaft?

$$T = \frac{3000 \times 63{,}000}{120} = 15{\cdot}75 \times 10^5 \text{ in. lbs.}$$

$$T = \frac{\pi}{16}\left(\frac{D^4 - d_1^4}{D}\right) f_s$$

$$\therefore \quad 15{\cdot}75 \times 10^5 = \frac{\pi}{16}\left(\frac{12^4 - 9^4}{12}\right) f_s$$

$$= \frac{\pi}{16} \times 1181 \times f_s$$

$$\therefore \quad f_s = \frac{15{\cdot}75 \times 10^5 \times 16}{\pi \times 1181}$$

$$= \mathbf{6800\ lb./in.^2}$$

Example 3.—A motor car shaft is of tubular form, $1\frac{1}{4}''$ internal dia. and $\frac{1}{4}''$ thick. The engine develops 15 H.P. at 2000 R.P.M. Determine the stress in the tube at the inner and outer dia. when full torque is transmitted through 4 to 1 gearing.

$$\text{At engine,} \quad T = \frac{15 \times 63{,}000}{2000} = 472{\cdot}5 \text{ in. lbs.}$$

Gear ratio 4 : 1.

$$\therefore \quad \text{At shaft, } T = 4 \times 472{\cdot}5 = 1890 \text{ in. lbs.}$$

$$\frac{T}{J} = \frac{f_s}{r}$$

For a thin circular tube,

$$J = 2\,\pi\,R^3\,t$$

where R = mean radius of tube.
t = thickness of tube.

hence $J = 2\,\pi \times {\cdot}75^3 \times {\cdot}25 = {\cdot}664 \text{ in.}^4$

$\therefore$ For **outer** surface of tube ($r = {\cdot}875''$)

$$\frac{1890}{{\cdot}664} = \frac{f_s}{{\cdot}875}$$

$$\therefore \quad f_s \text{ (max.)} = \frac{1890 \times {\cdot}875}{{\cdot}664} = 2490 \text{ lb./in.}^2$$

For **inner** surface of tube ($r = {\cdot}625''$)

$$\frac{1890}{{\cdot}664} = \frac{f_s}{{\cdot}625}$$

$$f_s \text{ (min.)} \quad \frac{1890 \times {\cdot}625}{{\cdot}664} = 1780 \text{ lb./in.}$$

Example 4.—(*a*) Find the horse-power transmitted by a shaft 8″ dia. running at 120 R.P.M. Let $f_s = 8000$ lb./in.². (*b*) The connecting coupling is the flanged type forged solid with the shaft, and has six fitted bolts $2\frac{1}{8}$″ dia.

What is the average shear stress in the bolts, assuming the load is evenly distributed?

(*a*)
$$T = \frac{\pi d^3}{16} f_s$$

$$= \frac{\pi}{16} \times 8^3 \times 8000 = 8{\cdot}04 \times 10^5 \text{ in. lbs.}$$

$$\therefore \quad HP = \frac{T \times N}{63{,}000}$$

$$= \frac{8{\cdot}04 \times 10^5 \times 120}{63{,}000} = \mathbf{1530}$$

(*b*) To determine bolt P.C. dia.

Say bolts are turned from 2″ hex. bar and reduced at end to take a standard 2″ nut,

Width across corners of nut = $3\frac{3}{4}$″ approx.
Allowing $\frac{5}{8}$″ radius in shaft at flange.
∴ Bolt P.C. dia. = $8 + 3\frac{3}{4} + 1\frac{1}{4}'' = 13''$
Twisting moment = $8{\cdot}04 \times 10^5$ in. lbs.

Resisting moment of bolts = $n \times a \times R \times S$

where n = Number of effective bolts = 6 say.
a = Area of one bolt = $3{\cdot}54$ in.²
R = Radius of bolt circle = $6\frac{1}{2}$″
S = Average shear stress in bolts.

$$\therefore \quad 8{\cdot}04 \times 10^5 = 6 \times 3{\cdot}54 \times 6{\cdot}5 \times S$$

$$\therefore \quad S = \frac{8{\cdot}04 \times 10^5}{6 \times 3{\cdot}54 \times 6{\cdot}5} = \mathbf{5820 \text{ lb./in.}^2}$$

Note.—The assumption is made here that all six bolts take the shear, but this is seldom achieved in practice. Also, the bolts are subject to bending effects not allowed for. Hence, using above expression, with bolts of the same material as the shaft, the bolt stress should be kept about half the shaft stress.

In this example, the bolts could be made of a higher carbon steel than the shaft. As the bolts are somewhat large in proportion, however, and entail considerable fitting, probably a better method for a shaft of this size would be to insert a face key between the coupling faces. This would relieve the bolts of shear stresses and their size could then be reduced and ordinary turned bolts used.

Section II.

BEAMS AND BENDING.

Formulae.

$$M = \frac{fI}{y} = fZ$$

where M = Max. bending moment (in tons).
I = Greatest moment of inertia about neutral axis (in.^{4}).
f = Extreme fibre stress (tons/in.^{2}).
y = Distance from neutral axis to extreme fibre (ins.).
Z = Modulus of section for bending (in.^{3}).

(Note that M is sometimes written as BM).

Theory of Parallel Axis.

To find the moment of inertia of a section about an axis XX parallel to axis CG passing through the centroid of the section.

$$I_{xx} = Ah^2 + I_{CG}$$

where I_{xx} = Moment of inertia about XX the parallel axis.
I_{CG} = Moment of inertia of section about CG.
A = Area of section.
h = Perpendicular distance between XX and CG.

Values of I for a particular section may either be calculated or, in the case of a standard section, be found direct from British Standard Tables.

Loaded Beams—Standard Cases.

Case I. *Cantilever with concentrated load at end.*

$$M = Wl$$

Case II. *Cantilever with uniformly distributed load.*

$$M = \frac{Wl}{2}$$

Case III. *Beam supported at two points with single load at centre of span.*

$$M = \frac{Wl}{4}$$

Case IV. *Beam supported at two points with uniformly distributed load along its length,*

$$M = \frac{Wl}{8}$$

where W = Total load (tons).
l = Effective span (ins.)

Example 5.—A M.S. pump lever rocking shaft is shown in Fig. 1. For the loading indicated, draw the bending moment diagram and calculate the diameter at the centre portion of the shaft. Take the fibre stress as 4 tons/in.^{2}.

Referring to Fig. 1. To find reactions R_A and R_B at bearings Take moments about R_A.

$$\text{Then } 46 \times R_B = 9 \times 2{\cdot}75 + 35 \times 3{\cdot}5$$
$$= 24{\cdot}75 + 122{\cdot}5 = 147{\cdot}25$$
$$R_B = \frac{147{\cdot}25}{46} = \mathbf{3{\cdot}2\ tons.}$$
$$R_A = (2{\cdot}75 + 3{\cdot}5) - 3{\cdot}2$$
$$= 6{\cdot}25 - 3{\cdot}2 = \mathbf{3{\cdot}05\ tons.}$$

To draw B.M. diagram (Fig. 2).

At $2\frac{3}{4}$ ton load, B.M. $= R_A \times 9 = 3{\cdot}05 \times 9 = 27{\cdot}45$ in. tons.
„ $3\frac{1}{2}$ „ „ B.M. $= R_B \times 11 = 3{\cdot}2 \times 11 = 35{\cdot}2$ in. tons.

From diagram, it is seen that the maximum bending moment on the centre portion of the shaft occurs slightly to left of $3\frac{1}{2}$ tons load—say 12″ from R_B without allowing for a radius at step up to larger diam.

$$\therefore \text{ Max. B.M. on centre portion } = (12'' \times 3{\cdot}2) - (1 \times 3{\cdot}5)$$
$$= 38{\cdot}4 - 3{\cdot}5 = \mathbf{34{\cdot}9\ in.\ tons.}$$
$$\text{B.M.} = fZ$$
$$\therefore \quad Z = \frac{34{\cdot}9}{4} = 8{\cdot}72 \text{ in.}^3$$

From standard tables read off equivalent shaft dia. for above modulus.

Hence shaft dia. = **$4\frac{1}{2}$″**

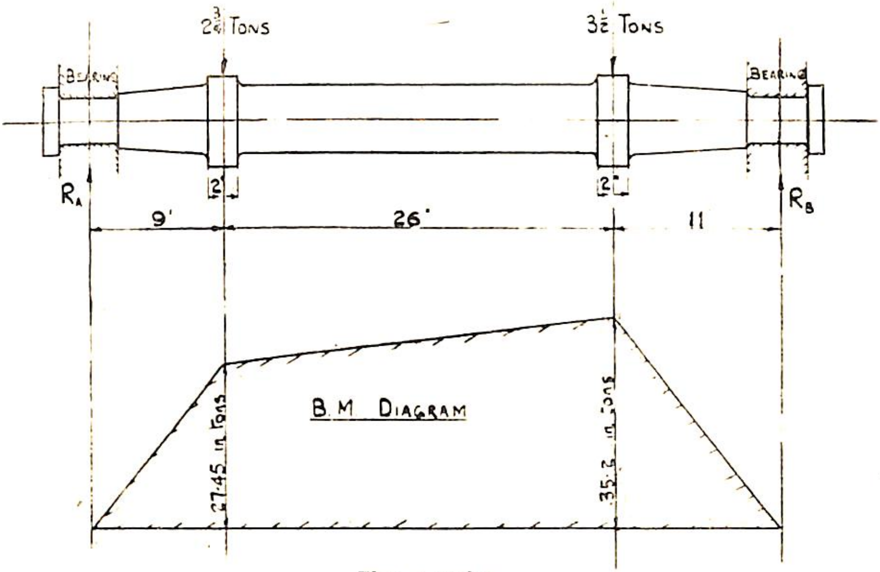
Example 6.—A cast iron beam (Fig. 3) has to carry a distributed load of 15 tons over a span of 10 feet. Calculate the max. tensile stress in the bottom flange and the max. compressive stress in the top flange.

To find position of XX (neutral axis).

Taking moments about ZZ and setting out figures in tabular form.

PART.	Area A. in.^{2}	d ins.	$A \times d$
Bottom Flange,	24	1	24
Web,	15	7	105
Top Flange,	6	$12\frac{3}{4}$	76·5
	Σ 45 in.^{2}		Σ 205·5 in.^{3}

d = distance from ZZ to centre of area.



Figs. 1 and 2.

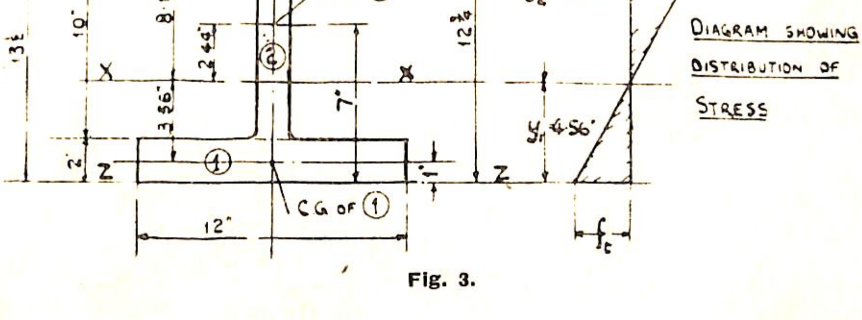


Fig. 3.

$$\therefore \quad y_1 = \frac{\Sigma A d}{\Sigma A} = \frac{205 \cdot 5}{45} = 4 \cdot 56''$$

$$\therefore \quad y_2 = 13 \cdot 5 - 4 \cdot 56 = 8 \cdot 94''$$

To find total I_{xx} of Section.

$$\text{For Bottom Flange, } I_{CG} = \frac{b\,d^3}{12} = \frac{12 \times 2^3}{12} = 8 \text{ in.}^4$$

$$\text{For Web, } I_{CG} = \frac{b\,d^3}{12} = \frac{1 \cdot 5 \times 10^3}{12} = 125 \text{ in.}^4$$

$$\text{For Top Flange } I_{CG} = \frac{b\,d^3}{12} = \frac{4 \times 1 \cdot 5^3}{12} = 1 \cdot 125 \text{ in.}^4$$

From the theory of parallel axis, the I_{xx} of each part is now calculated, the summation giving the total I_{xx}.

Putting the figures in tabular form.

PART.	Area A.	h	A h^2	I_{CG}	$I_{CG}+Ah^2$
Bottom Flange,	24	3·56	304	8	312
Web	15	2·44	89·5	125	214·5
Top Flange,	6	8·19	402	1·125	403·125

$$\therefore \quad \text{Total } I_{xx} = 929 \cdot 625 \text{ in.}^4$$

For the conditions of loading,

$$M = \frac{Wl}{8} = \frac{15 \times 10 \times 12}{8}$$

$$= 225 \text{ in. tons.}$$

$$f = \frac{M \times y}{I}$$

$\therefore$ For Bottom Flange,

$$f_{\text{tensile}} = \frac{M \times y_1}{I}$$

$$= \frac{225 \times 4 \cdot 56}{929 \cdot 625} = 1 \cdot 103 \text{ tons/in.}^2$$

$$= 2475 \text{ lb./in.}^2$$

For Top Flange,

$$f_{\text{compressive}} = \frac{M \times y_2}{I}$$

$$= \frac{225 \times 8{\cdot}94}{929{\cdot}625} = 2{\cdot}164 \text{ tons/in.}^2$$

$$= \mathbf{4855\ lb./in.^2}$$

Example 7.—Determine the cross-sectional dimensions of a plate web girder (Fig. 4) carrying three concentrated loads of 10, 15, and 10 tons at $\frac{1}{4}$, $\frac{1}{2}$, and $\frac{3}{4}$ of span respectively for a span of 40 ft. Take average stress in flanges as 7 tons/in.². Depth of girder between centres of flanges $\frac{1}{16}$th of span. Breadth $\frac{1}{40}$th of span. Take the weight of the girder as 300 lbs. per foot of span.

In this example, the effect of the angles and stiffeners is neglected. The flanges are designed to take the bending and the web to take the shear, which method gives a close approximation.

Fig. 5 shows the separate bending moment diagrams for the loads and the girder weight.

$$\text{Max. B.M.} = \text{B.M.}_c + \text{B.M.}_w$$

Reactions R_1 and R_2 are equal.

$$\text{or,} \quad R_1 = R_2 = \frac{35}{2} = 17\tfrac{1}{2} \text{ tons.}$$

$$\therefore \quad \text{B.M.}_c = (17{\cdot}5 \times 20) - (10 \times 10)$$
$$= 350 - 100 = \mathbf{250\ ft.\ tons.}$$

$$\text{B.M.}_w = \frac{300}{2240} \times \frac{40 \times 40}{8} \qquad \left[\begin{array}{c}\text{Case IV.} \\ M = \dfrac{lW}{8}\end{array}\right]$$

$$= 26{\cdot}8 \text{ ft. tons.}$$

$$\therefore \quad \text{Max. B.M.} = 250 + 26{\cdot}8 = 276{\cdot}8 \text{ ft. tons.}$$
$$= \mathbf{3322\ in.\ tons.}$$

Referring to Fig. 4,

$$d = \tfrac{1}{16}\text{th of span} \quad \text{or } d = \frac{40}{16} = 2'\text{-}6'' = 30''$$

$$B = \tfrac{1}{40}\text{th of span,} \quad \text{or } B = \frac{40}{40} = 1 \text{ ft.} = 12''$$

$$\text{Max. B.M.} = \text{Moment of resistance of flange.}$$
$$= B \times t \times d \times f$$
$$\therefore \quad 3322 = 12 \times t \times 30 \times 7$$

$$\therefore \quad t = \frac{3322}{12 \times 30 \times 7} = 1{\cdot}32'' \text{ or } 1\tfrac{5}{16}''$$

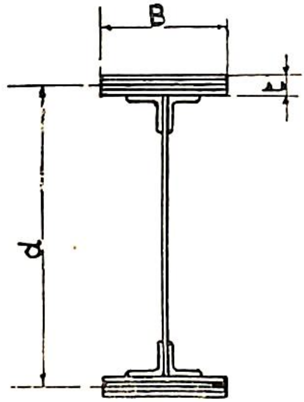

B
d

Fig. 4.

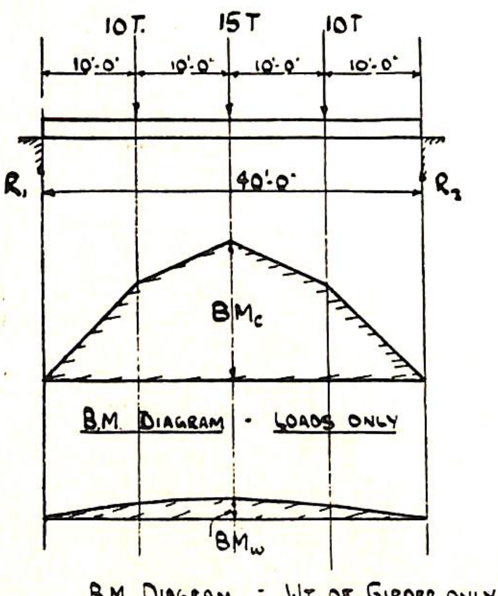

10T.
15T
10T
10'-0"
10'-0"
10'-0"
10'-0"
R₁
40'-0"
R₂
BMc
B.M. DIAGRAM - LOADS ONLY
BMw
B.M DIAGRAM - WT OF GIRDER ONLY

Fig. 5.

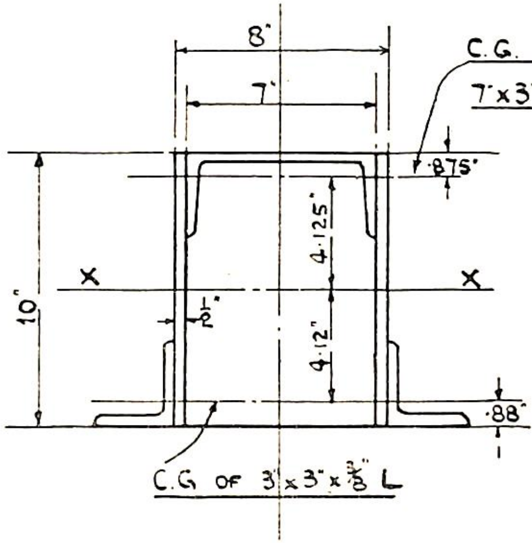

8"
7"
C.G. OF
7"x3" [
·875"
4·125"
X
X
10"
½"
4·12"
·88"
C.G OF 3"x3"x⅜" L

Fig. 6.

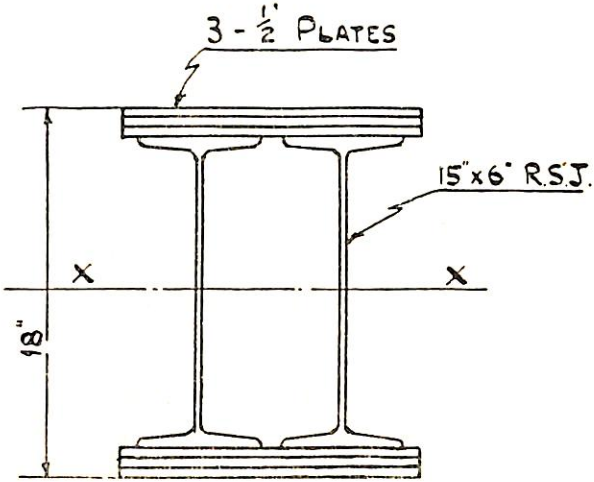

3 - ½" PLATES
15"x6" R.S.J.
X
X
18"

Fig. 7.

Assume plate thickness $= \frac{7}{16}''$

$\therefore$ No. of plates per flange $= \dfrac{1\frac{5}{16}''}{\frac{7}{16}''} = 3$

$\therefore$ Total girder depth $= 30'' + 1\frac{5}{16}'' = 31\frac{5}{16}''$

Web Thickness.

Max. shear at end $= 17{\cdot}5 + \dfrac{300 \times 20}{2240}$ (half girder weight).

$= 17{\cdot}5 + 2{\cdot}68 = 20{\cdot}18$ tons.

Average shear stress $= 2$ tons/in.^{2} say.

$\therefore$ Area of web $= \dfrac{20{\cdot}18}{2} = 10{\cdot}09$ in.^{2}

Length of web $= 30 - 1\frac{5}{16} = 28\frac{11}{16}''$

$\therefore$ Thickness of web $= \dfrac{10{\cdot}09}{28\frac{11}{16}''} = {\cdot}352$ say $\frac{3}{8}''$

Note.—For economy, the flange plates may be curtailed towards the supports, as the bending moment on the girder becomes less.

Example 8.—A box girder 10″ deep is built up of a channel, 2 side plates and 2 angles (Fig. 6). The neutral axis passes through the centre of the girder, which is uniformly loaded over a span of 20 ft., allowing a stress of 5 tons/in.^{2} in the material. Find the maximum load the girder will carry.

From tables in Steel Section Book,

I for $7'' \times 3''$ channel $= 3{\cdot}25$ in.^{4}
Area of $7'' \times 3''$ channel $= 4{\cdot}18$ in.^{2}
I for one $3'' \times 3'' \times \frac{3}{8}''$ angle $= 1{\cdot}72$ in.^{2}
Area of one $3'' \times 3'' \times \frac{3}{8}''$ angle $= 2{\cdot}11$ in.^{2}

I for one side plate $= \dfrac{{\cdot}5 \times 10^3}{12}$

$= 41{\cdot}6$ in.^{4}

To find I_{xx} for girder.

PART.	Area A.	h	A h^2	I_{CG}	$I_{CG}+A\,h^2$
Channel,	4·18	4·125	71·11	3·25	74·36
Two Plates,	10	0	0	83·2	83·2
Two Angles,	4·22	4·12	71·6	3·44	75·04

Total $I_{xx} = 232{\cdot}6$ in.^{4}

$$M = \frac{f \times I}{y}$$

$$= \frac{5 \times 232 \cdot 6}{5} = 232 \cdot 6 \text{ in. tons.}$$

$$232 \cdot 6 = \frac{Wl}{8} = \frac{W \times 20 \times 12}{8}$$

$$\text{Max. load } W = \frac{232 \cdot 6 \times 8}{20 \times 12} = \mathbf{7 \cdot 75 \text{ tons.}}$$

Example 9.—A compound girder of cross-section shown in Fig. 7, has to carry a uniformly distributed load of 75 tons over a 25-ft. span. The max. stress must not exceed 6 tons/in.^{2}. Calculate the breadth of the flange plates.

From tables,

I_{xx} for one $15'' \times 6''$ Joist $= 491 \cdot 9$ in.^{4}

For plates in top flange. Plates $\frac{1}{2}''$ thick.

Total flange thickness $= 1\frac{1}{2}'' = d$

$\therefore$ Distance from CG of flange to neutral axis of girder

$= 9'' - \frac{3}{4}'' = 8\frac{1}{4}'' = h.$

$$I_{xx} = I_{CG} + A h^2$$

$$= \frac{b d^3}{12} + b d h^2$$

$$= b \left(\frac{d^3}{12} + dh^2\right)$$

$$= b \left(\frac{1 \cdot 5^3}{12} + 1 \cdot 5 \times 8 \cdot 25^2\right)$$

$$= b\,(\cdot 28 + 102 \cdot 1)$$

$$= 102 \cdot 38\, b \text{ in.}^4$$

i.e., for one flange $I_{xx} = 102 \cdot 38\, b$ in.^{4}

$\therefore$ Total I_{xx} for girder $= 2 \times 491 \cdot 9 + 2 \times 102 \cdot 38\, b$

$= 983 \cdot 8 + 204 \cdot 76\, b$

$$M = \frac{f \times I}{y}$$

For uniformly-distributed load,

$$M = \frac{Wl}{8} = \frac{75 \times 25 \times 12}{8} = \mathbf{2815 \text{ in. tons.}}$$

$f = 6$ tons/in.^{2}

$y = 9''$

$$\therefore \quad 2815 = \frac{6\,(983{\cdot}8 + 204{\cdot}76\,b)}{9}$$

$$\therefore \quad 5902{\cdot}8 + 1228{\cdot}56\,b = 25335$$

$$\therefore \quad b = \frac{25335 - 5902{\cdot}8}{1228{\cdot}56} = 15{\cdot}85'', \text{ say } \mathbf{16''}$$

SECTION III.

COMBINED TWISTING AND BENDING.

Formulae.

$$M = fZ$$
$$T = f_s Z_t$$
$$q = \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2}$$
$$p = \frac{f}{2} + \sqrt{\left(\frac{f}{2}\right)^2 + f_s^2} = \frac{f}{2} + q$$

For solid circular shafts,

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi\, d^3}{16}\, q \quad \text{(Guest formula).}$$

Notation.

M = Max. bending moment (in. lbs.).
T = Max. twisting moment (in. lbs.).
T_e = Equivalent twisting moment due to combined twisting and bending (in lbs.).
Z = Modulus of section for bending (in.^{3}).
Z_t = Modulus of section for twisting (in.^{3}).
f = Extreme fibre stress due to bending (lb./in.^{2}).
f_s = Max. shear stress due to twisting and acting on same plane as f (lb./in.^{2}).
q = Max. shear stress due to combined bending and twisting (lb./in.^{2}).
d = Dia. of shaft (ins.).
p = Max. principal normal stress (lb./in.^{2}).

Notes on above.

The formula for the equivalent bending moment for shafts has not been stated, since it enters very little into practice. In most cases of combined twisting and bending, the material selected

is some ductile material such as mild steel, which fails in shear. Hence for shafts for maximum strength the equivalent twisting moment should be used.

In cases where brittle materials such as cast iron are subject to combined twisting and bending, it is more important to determine the value of the principal stress p which is a tensile stress and is greater than the shear stress. Cast iron, of course, is slightly weaker in tension than in shear.

It should be mentioned also that the value f_s given in the q and p formulae, while correct for symmetrical sections such as shafts, is not quite true for a rectangular section. The values found, however, will be slightly higher than the actual, hence keeping on the safe side.

The permissible shear stress q should not exceed half the permissible tensile stress for the material.

Example 10.—Part of a spur reduction gear is shown in Fig. 8. The pinion has 20 teeth, 4 D.P. teeth machine cut 20° involute, and transmits 60 H.P. at 800 R.P.M. The reduction ratio is 5 to 1. Calculate the diameter of pinion shaft and wheel shaft, taking the maximum permissible shear stress as 8000 lb./in.^{2}

$$\text{P.C.D. of pinion} = \frac{n}{\text{D.P.}} = \frac{20}{4} = 5''$$

$$T = \frac{\text{H.P.} \times 63{,}000}{N}$$

$$= \frac{60 \times 63{,}000}{800} = \mathbf{4725\ in.\ lbs.}$$

$$\therefore \quad \text{Tangential tooth load } E = \frac{\text{Torque}}{\text{Radius of pitch circle}}$$

$$= \frac{4725}{2{\cdot}5} = \mathbf{1890\ lbs.}$$

$$\text{Involute angle } \alpha = 20^\circ$$

$$\therefore \quad \text{Max. tooth load } P = \frac{E}{\cos 20^\circ} = \frac{1890}{{\cdot}9397} = 2010 \text{ lbs.}$$

The force E is the driving force and puts a torque on the shafts, while P produces bending effects (see Fig. 8a).

For Pinion Shaft.

Twisting moment T = 4720 in. lb.
Bending moment M = 2010 × 7½
= 15075 in. lbs.

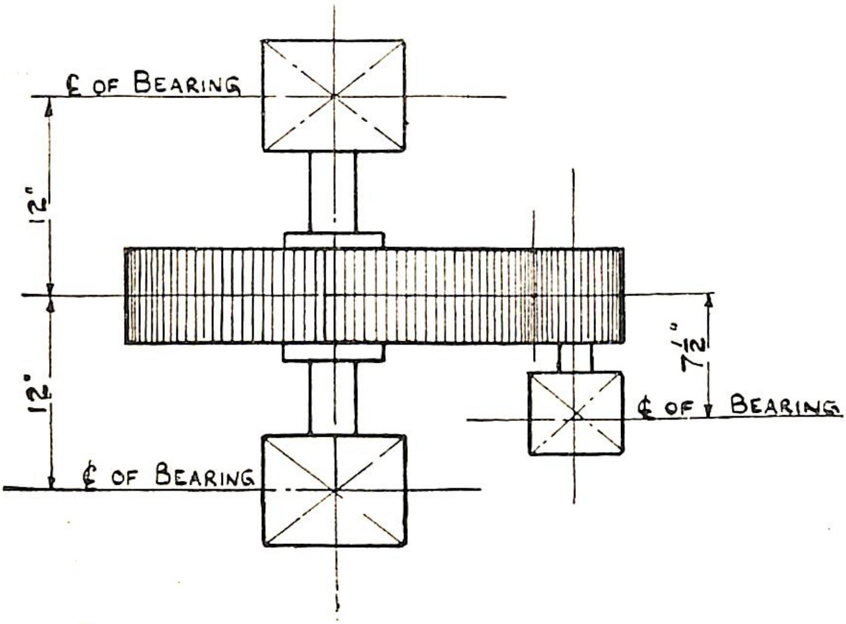


Fig. 8.

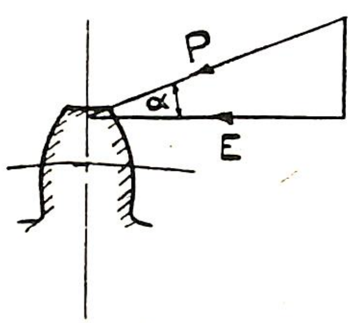


Fig. 8a.

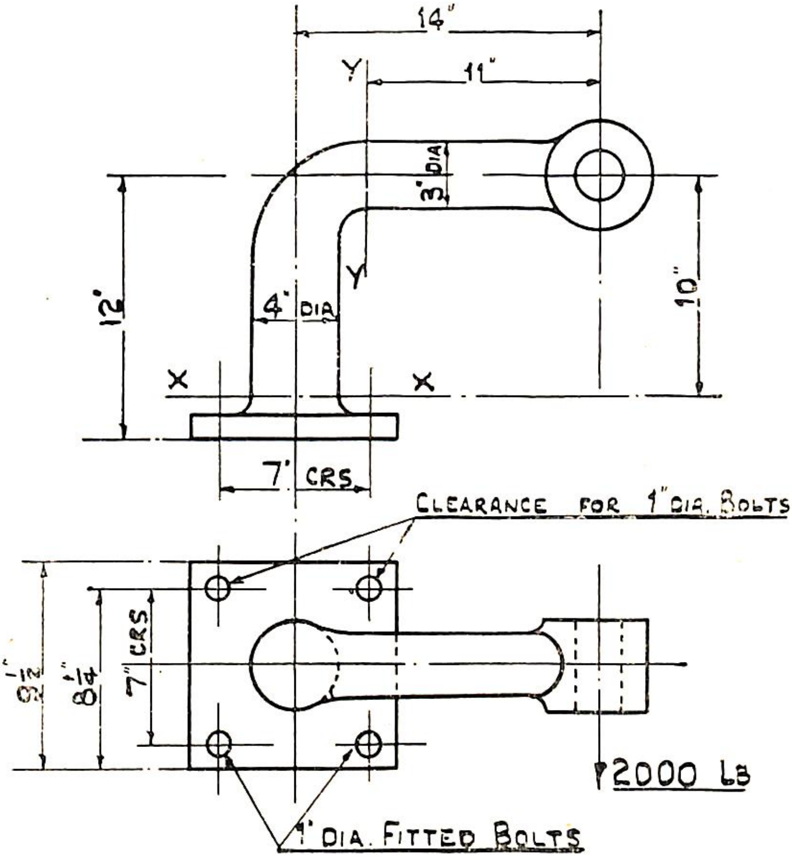

14"
11"
Y
Y
3" DIA
12"
10"
4" DIA
X
X
7" CRS
CLEARANCE FOR 1" DIA. BOLTS
9½"
8¼"
7" CRS
2000 LB
1" DIA. FITTED BOLTS

Fig. 9.

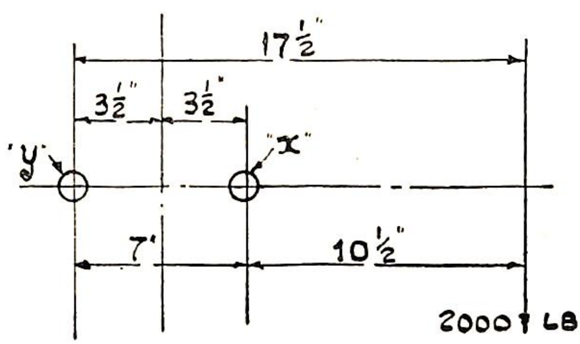

17½"
3½"
3½"
"y"
"x"
7"
10½"
2000 LB

Fig. 9a.

$$
\begin{aligned}
T_e &= \sqrt{M^2 + T^2} \\
&= 1000\sqrt{15{\cdot}075^2 + 4{\cdot}72^2} \\
&= 1000\sqrt{227 + 22{\cdot}3} \\
&= 1000\sqrt{249{\cdot}3} \\
&= 15{,}800 \text{ in. lb.}
\end{aligned}
$$

$q = 8000$ lb./in.^{2}

$$\therefore \quad 15800 = \frac{\pi}{16} d^3 \times 8000$$

$$\therefore \quad d^3 = \frac{15800 \times 16}{\pi \times 8000} = 10{\cdot}04$$

$$\therefore \quad d = 2{\cdot}15$$ **say $2\frac{1}{4}''$ dia.**

For Wheel Shaft.

Gear ratio, 5 to 1.

$\therefore$ Twisting moment $= 4720 \times 5 = 23600$ in. lb.

Bending moment $= \dfrac{2010 \times 24}{4} = 12060$ in. lb.

(The form $\dfrac{W\,L}{4}$ is used here, and gives a higher B.M. than the actual and hence a stronger shaft. The reactions are assumed to act at the centre of the bearing. Some designers take reaction at $\frac{1}{3}$ L where L is the bearing length, this giving a closer span and a smaller M).

$$
\begin{aligned}
\therefore \quad T_e &= 1000\sqrt{12{\cdot}06^2 + 23{\cdot}6^2} \\
&= 1000\sqrt{145{\cdot}2 + 556} \\
&= 1000\sqrt{701{\cdot}2} = \mathbf{26500 \text{ in. lb.}}
\end{aligned}
$$

$$\therefore \quad 26500 = \frac{\pi}{16} d^3 \times 8000$$

$$d^3 = \frac{26500 \times 16}{\pi \times 8000} = 16{\cdot}9$$

$$\therefore \quad d = 2{\cdot}56$$ **say $2\frac{5}{8}''$**

Example 11.—Fig. 9 shows a solid forged steel bracket for holding one end of a tie bar subjected to a load of 2000 lbs. Calculate the stress due to bending across the plane YY, the stress due to combined bending and twisting across plane XX and the stresses in the flange bolts resisting the twisting, sliding and toppling action of the load. State the nature of the stresses in each case.

Section YY.

Bending moment $= 2000 \times 11 = 22000$ in. lbs.

$$M = fZ \quad (Z = \frac{\pi}{32} d^3 \text{ for circular section}).$$

$$\therefore \quad 22000 = \frac{\pi \times 3^3}{32} \times f$$

$$f = \frac{22000 \times 32}{\pi \times 27} = \mathbf{8300\ lb./in.^2}$$

f is tensile on one side and compressive on the other.

Section XX.

Bending moment $= 2000 \times 10 = 20000$ in. lbs.
Twisting moment $= 2000 \times 14 = 28000$ in. lb.

$$\begin{aligned} T_e &= \sqrt{M^2 + T^2} \\ &= 10000 \sqrt{2^2 + 2{\cdot}8^2} \\ &= 10000 \sqrt{4 + 7{\cdot}84} \\ &= 10000 \sqrt{11{\cdot}84} \qquad = \mathbf{34400\ in.\ lbs.} \end{aligned}$$

$$T_e = \frac{\pi d^3}{16} \times q$$

$$34400 = \frac{\pi}{16} \times 4^3 \times q$$

$$\therefore \quad q = \frac{34400 \times 16}{64 \times \pi} \qquad = \mathbf{2740\ lb./in.^2}$$

This is the max. shear stress across XX, due to combined twisting and bending.

Stresses on Bolts.

Twisting and sliding on bolts produces shear. Take the two fitted bolts only as resisting this effect. Referring to Fig. 9 (a),

$$\text{Load on bolt } y = \frac{2000 \times 10\frac{1}{2}}{7} = 3000 \text{ lb.}$$

Load on bolt $x = 3000 + 2000 = 5000$ lb.
Area of 1″ bolt $= {\cdot}7854$ in.^{2}

$$\therefore \quad \text{Shear stress on bolt } y = \frac{3000}{{\cdot}7854} = 3820 \text{ lb./in.}^2$$

$$\text{Shear stress on bolt } x = \frac{5000}{{\cdot}7854} = 6355 \text{ lb./in.}^2$$

Toppling action—take two black bolts only.

$$\text{Overturning moment} = 2000 \times 12 = 24000 \text{ in. lb.}$$
$$\text{Righting moment} = P \times 8\tfrac{1}{4}$$
$$\text{Hence } P = \text{load on bolts} = \frac{24000}{8\frac{1}{4}} = 2910 \text{ lb.}$$
$$\therefore \quad \text{Load per bolt} = \frac{2910}{2} = 1455 \text{ lb.}$$
$$\text{Area at bottom of thread of } 1'' \text{ bolt} = \cdot 554 \text{ in.}^2$$
$$\therefore \quad \text{Tensile stress in bolts} = \frac{1455}{\cdot 554} = 2630 \text{ lb./in.}^2$$

Example 12.—The bent M.S. lever A, shown on Fig. 10 fulcrums about B. The pin C is fitted to the lever and carries a link with an inclined pull P of 1000 lbs. This is obtained from an applied load W inclined as shown in the end view of the roller. Find W, and determine the stresses at XX and YY, due to the bending and twisting action of the loads. State the nature of the stress in each case.

$$P \cos 30^\circ \times 5 = W \cos 30^\circ \times 12$$
$$\therefore \quad 1000 \times 5 = W \times 12$$
$$\therefore \quad W = \frac{1000 \times 5}{12} = 416 \text{ lbs.}$$

Consider Section YY.

This is subject to twisting and bending due to W.

W has two components.

Vertical component V_c produces twisting and bending in lever.

Horizontal component H_c produces pure bending.

$$V_c = \cos 30^\circ \times 416 = \cdot 866 \times 416 = 360 \text{ lbs.}$$
$$H_c = \sin 30^\circ \times 416 = \cdot 5 \times 416 = 208 \text{ lbs.}$$

Consider H_c.

Bending arm $= 10{\cdot}5''$. Section $3''$ wide $\times \frac{3}{4}''$ deep.

M = bending arm $\times H_c = 10{\cdot}5 \times 208 = 2184$ in. lbs.

$$Z = \frac{bd^2}{6} = \frac{3 \times \cdot 75^2}{6} = \cdot 281 \text{ in.}^2$$

Now $M = f_1 Z$,

$$\therefore \quad f_1 = \frac{2185}{\cdot 281} = 7790 \text{ lb./in.}^2 \text{ on the long side.}$$

f_1 is both tensile and compressive and acts at right angles to the direction of loading, as in an ordinary beam.

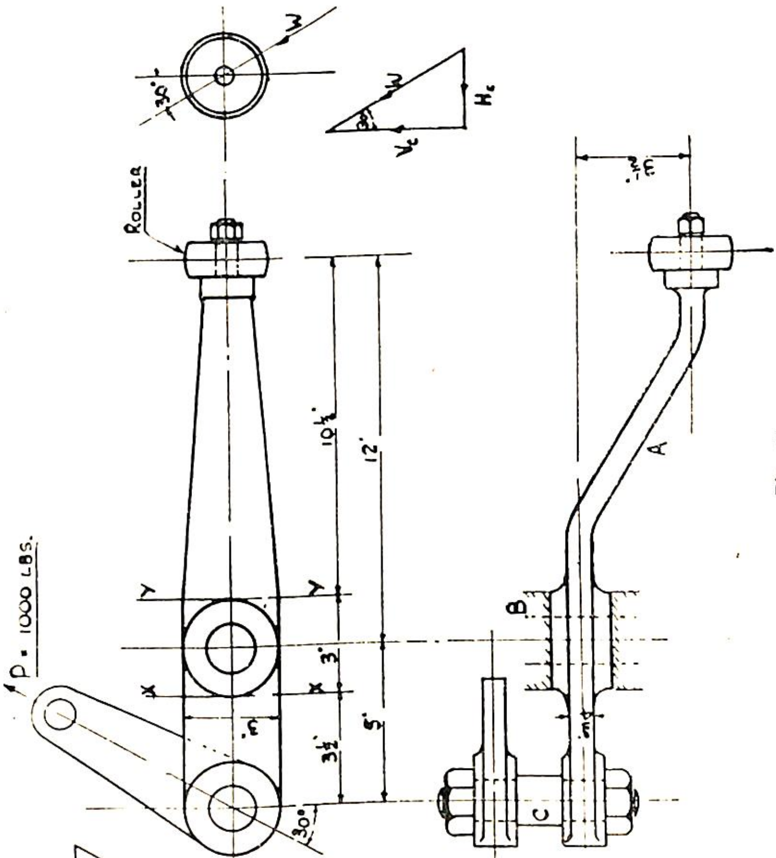


Fig. 10

Consider V_c.

Due to combined twisting and bending, the main stress to determine is the max shear stress q.

For bending,

$$M = \text{bending arm} \times V_c = 10{\cdot}5 \times 360 = 3780 \text{ in. lbs.}$$

$$Z = \frac{bd^2}{6} = \frac{{\cdot}75 \times 3^2}{6} = 1{\cdot}125 \text{ in.}^3$$

Now $M = f\,Z$,

$$\therefore \quad f = \frac{3780}{1{\cdot}125} = 3360 \text{ lb./in.}^2$$

For twisting,

$$T = \text{twisting arm} \times V_c = 3{\cdot}5 \times 360 = 1260 \text{ in. lb.}$$

$$Z_t = {}^2/_9\, B\, b^2 \text{ (approx.) for rectangle.}$$

$$= {}^2/_9 \times 3 \times {\cdot}75^2 = {\cdot}375 \text{ in.}^3$$

Now $T = f_s\, Z_t$,

$$\therefore \quad f_s = \frac{1260}{{\cdot}375} = 3360 \text{ lb./in.}^2 \text{ at the middle of the long side.}$$

$$q = \sqrt{\left(\frac{f_1}{2}\right)^2 + f_s^2}$$

$$q = \sqrt{\left(\frac{7790}{2}\right)^2 + 3360^2}$$

$$= 1000 \sqrt{3{\cdot}9^2 + 3{\cdot}36^2}$$

$$= 1000 \sqrt{15{\cdot}2 + 11{\cdot}3}$$

$= 5140$ lbs./sq. in. Maximum shear stress at the middle of the long side.

Principal stress $\quad P = \dfrac{7790}{2} + 5140$

$$= 9040 \text{ lbs./sq. in.}$$

Now Consider Section XX.

Pull P resolves into vertical and horizontal components.

V_c produces twisting and bending.

H_c gives a case of eccentric loading producing bending and thrust in the lever.

$$H_c = \sin 30^\circ \times 1000 = 500 \text{ lbs.}$$

$$V_c = \cos 30^\circ \times 1000 \times 866 \text{ lbs.}$$

Consider H_c. (For notation connected with this, see Section V.).

$$\text{Max. stress (Compressive)} \quad f_c = \frac{W_e}{A}\left(1 + \frac{xy_1}{k^2}\right)$$

$$W_e = H_c = 500 \text{ lbs.} \qquad x = 2{\cdot}75''$$
$$A = 3 \times {\cdot}75 = 2{\cdot}25 \text{ in.}^2 \qquad y_1 = {\cdot}375''$$

$$k^2 = \frac{I}{A} \quad \left(I = \frac{bd^3}{12} = \frac{3 \times {\cdot}75^3}{12} \doteq {\cdot}105 \text{ in.}^4\right)$$

$$= \frac{{\cdot}105}{2{\cdot}25} = {\cdot}0467 \text{ in.}^2$$

$$\therefore \quad f_c = \frac{500}{2{\cdot}25}\left(1 + \frac{2{\cdot}75 \times {\cdot}375}{{\cdot}0467}\right)$$
$$= 222 \quad (1 + 22{\cdot}05)$$
$$= 222 \times 23{\cdot}05 = \mathbf{5120 \text{ lb./in.}^2}$$

The tensile stress due to this eccentricity would be

$$f_t = 222 \quad (1 - 22{\cdot}05)$$
$$= 222 \times -21{\cdot}05 = -\mathbf{4675 \text{ lb./in.}^2} \text{ on the long side.}$$

The negative sign merely denotes the opposite kind of stress.

Consider V_c.

For bending,

$$M = \text{bending arm} \times V_c = 3{\cdot}5 \times 866 = 3030 \text{ in. lbs.}$$
$$Z = 1{\cdot}125 \text{ in.}^3 \text{ as before.}$$

Now $M = f Z$

$$\therefore \quad f = \frac{3030}{1{\cdot}125} = 2700 \text{ lb./in.}^2$$

For twisting,

$$T = \text{twisting arm} \times V_c = 2{\cdot}75 \times 866 = 2380 \text{ in. lb.}$$
$$Z_t = {\cdot}375 \text{ in.}^3 \text{ as before.}$$

Now $T = f_s Z_t$

$$f_s = \frac{2380}{{\cdot}375} = 6350 \text{ lb./in.}^2 \text{ at the middle of the long side.}$$

$$\therefore \quad q = \sqrt{\left(\frac{f_t}{2}\right)^2 + f_s^2}$$

$$q = \sqrt{\left(\frac{4675}{2}\right)^2 + 6350^2}$$

$$= 1000 \sqrt{2{\cdot}34^2 + 6{\cdot}35^2}$$
$$= 1000 \sqrt{5{\cdot}46 + 40{\cdot}3}$$
$$= 1000 \sqrt{45{\cdot}76}$$
$$= 6780 \text{ lbs./sq. in.}$$

Maximum shear stress at the middle of the long side.

$$\text{Principal shear P} = \frac{4675}{2} + 6780$$
$$= 9120 \text{ lbs./sq. in.}$$

Section IV.

COLUMNS AND STRUTS.

Formulae.

Rankine-Gordon formula for ordinary columns,

$$P_r = \frac{A f_c}{1 + a \left(\frac{l}{k}\right)^2}$$

where P_r = crushing or crippling load (tons).
f_c = direct crushing stress (tons/in.^{2}).
A = area of cross section of column (ins.^{2})
l = length of column (ins.).
k = *least* radius of gyration (ins.).
a = constant.

$$k = \sqrt{\frac{I}{A}}$$

where I = *least* moment of inertia of section.

Usual values for f_c and a are given in Table 4 in Appendix. These vary for different materials and end fixings.

$$\text{The safe load P} = \frac{P_r}{\text{Factor of safety}}$$

For long columns, Euler's formula applies.

$$P_e = \frac{\pi^2 E I}{l^2} \text{ for hinged or rounded ends.}$$

$$= \frac{\pi^2 E I}{4 l^2} \text{ for one end fixed.}$$

where E = modulus of elasticity (tons/in.^{2})

For short columns, where buckling effects are absent, and hence material is in direct compression, these equations reduce to

$$P_r = P_e = A f_c$$

Note.—In general, a column or pillar is fixed at both ends and is vertical. Struts may be inclined, one or both ends fixed rigidly, or one or both ends pin-jointed or hinged.

Example 13.—A strut in a framed structure is made of a steel pipe 6″ outside dia. and $\frac{1}{2}$″ thick. The length is 10 feet, and the pipe is pin-jointed at both ends. With a factor of safety of 5, what is the safe load?

$$\text{Crippling load} \quad P_r = \frac{A f_c}{1 + a \times \left(\frac{l^2}{k^2}\right)}$$

$$A = (\text{area } 6'' \text{ dia.} - \text{area } 5'' \text{ dia.})$$
$$= 28{\cdot}27 - 19{\cdot}63 = 8{\cdot}64 \text{ in.}^2$$
$$f = 21 \text{ tons/in.}^2 \text{ (from Table 4).}$$
$$a = \frac{1}{7500} \text{ for hinged ends.}$$
$$l = 10 \times 12 = 120 \text{ ins.}$$
$$k^2 = \frac{I}{A}$$

For a circular pipe, I (least) = I (greatest)

$$I = \frac{\pi}{64} (D^4 - d^4)$$
$$= \frac{\pi}{64} (6^4 - 5^4)$$
$$= \frac{\pi}{64} \times 671 = 32{\cdot}9 \text{ in.}^4$$

$$\therefore \quad k^2 = \frac{32{\cdot}9}{8{\cdot}64} = 3{\cdot}82 \text{ in.}^2$$

$$\therefore \quad P_r = \frac{8{\cdot}64 \times 21}{1 + \frac{1}{7500} \times \frac{120^2}{3{\cdot}82}}$$
$$= \frac{181{\cdot}5}{1 + {\cdot}505} = 120{\cdot}5 \text{ tons.}$$

$$\text{Factor of safety} = 5$$

$$\therefore \quad \text{Safe load} = \frac{121}{5} = \mathbf{24{\cdot}2 \text{ tons.}}$$

Example 14.—A column for a crane gantry consists of two 16" × 6" R.S. joists, connected by ½" plates, as shown on Fig. 11. The length is 27 feet and the ends may be considered as fixed. Calculate the safe load. Factor of safety = 5.

$$P_r = \frac{A f_c}{1 + a \times \left(\frac{l^2}{k^2}\right)}$$

From Section book, for 16" × 6" R.S.J.

Least moment of inertia = 22·47 in.^{4}
Area of joist = 14·71 in.^{2}

For the whole column, the least I is about *yy* axis.

To find I_{YY} for column. For 2 joists,

$$\begin{aligned} I_{YY} &= (I_{CG} + Ah^2) \times 2 \\ &= (22{\cdot}47 + 14{\cdot}71 \times 7{\cdot}5^2) \times 2 \\ &= 1700 \text{ in.}^4 \end{aligned}$$

The connecting plates are negligible and can be omitted from the moment of inertia.

Hence, I_{YY} for column = 1700 in.^{4}
Area of cross section = (2 × 14·71) + (22 × 1)
= 51·42 in.^{2}

$$\therefore \quad k^2 = \frac{1701}{51{\cdot}42} = 33{\cdot}1 \text{ in.}^2$$

$$l = 27 \times 12 = 324 \text{ ins.}$$

From Table 4,

$$a = \frac{1}{30{,}000} \text{ for ends fixed.}$$

$$f_c = 21 \text{ T/in.}^2 \text{ for steel.}$$

$$\therefore \quad P_r = \frac{51{\cdot}4 \times 21}{1 + \frac{1}{30{,}000} \times \frac{324^2}{33{\cdot}1}}$$

$$= \frac{1080}{1 + {\cdot}106}$$

$$= \frac{1080}{1{\cdot}106} = \textbf{977 tons.}$$

Factor of safety = 5

$$\therefore \quad \text{Safe load} = \frac{977}{5} = \textbf{195·4 tons.}$$

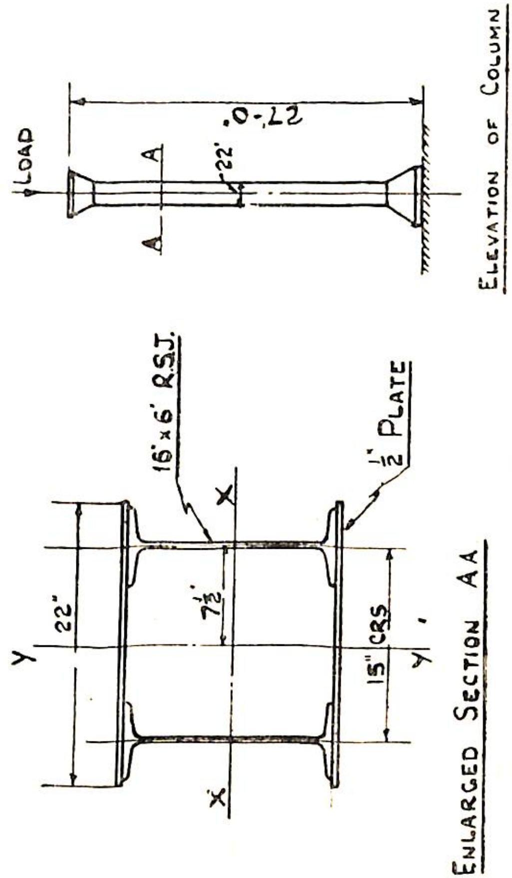

LOAD
A
A
22'
27'-0"
ELEVATION OF COLUMN
16"x6" R.S.J.
½" PLATE
X
X
Y
Y
22"
7½"
15" CRS
ENLARGED SECTION AA

FIG. 11.

SECTION V.

ECCENTRIC LOADING.

Formulae.

$$f_1 = \frac{W_e}{A}\left(1 + \frac{xy_1}{k^2}\right)$$

$$f_2 = \frac{W_e}{A}\left(1 - \frac{xy_2}{k^2}\right)$$

where f_1 = extreme fibre stress nearest load (tons/in.^{2}).
f_2 = extreme fibre stress remote from load (tons/in.^{2}).
W_e = actual eccentric load (tons).
A = area of section (in.^{2}).
x = arm of eccentricity (ins.).
y_1 = perpendicular distance from neutral axis of section to outside edge nearest load (ins.).
y_2 = perp. distance from neutral axis of section to outside edge remote from load (ins.).
k = radius of gyration (ins.).

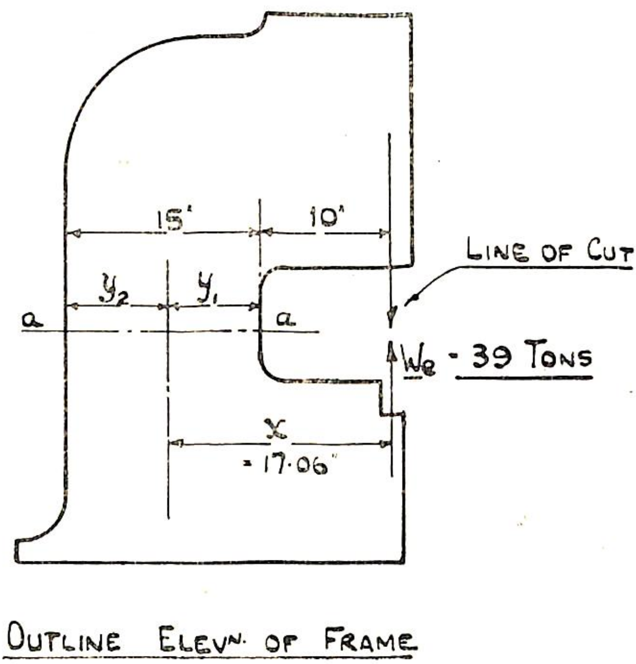


Fig. 12.

Example 15.—A shearing machine (Fig. 12) exerts a force of 39 tons on each frame. The cast iron frame section is shown on Fig. 13, the position of the centroid being already determined. Calculate the maximum tensile and compressive stresses on the section of the frame due to the given load.

First find I_{xx} of section.

As indicated in Section II., this is the summation of the I_{xx} for each separate part—(1), (2) and (3)—as shown on Fig. 13.

For each part, $I_{xx} = I_{CG} + Ah^2$ (theory of parallel axis).

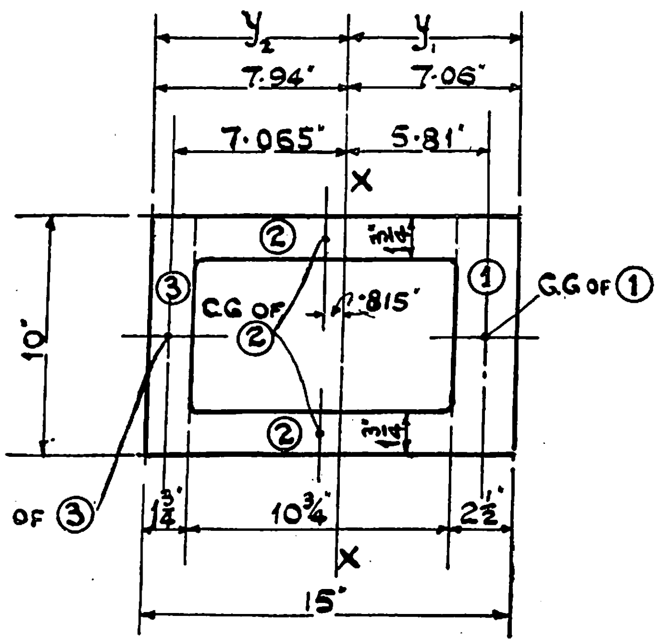
A = area of each part.

h = distance from C.G. of part to XX.

Part 1, $I_{CG} = \dfrac{bd^3}{12} = \dfrac{10 \times 2{\cdot}5^3}{12} = 13 \text{ in.}^4$

Part 2, $I_{CG} = \dfrac{bd^3}{12} = \dfrac{3{\cdot}5 \times 10{\cdot}75^3}{12} = 362{\cdot}5 \text{ in.}^4$

Part 3, $I_{CG} = \dfrac{bd^3}{12} = \dfrac{10 \times 1{\cdot}75^3}{12} = 4{\cdot}5 \text{ in.}^4$



ENLARGED SECTION aa

Fig. 13.

Setting out in tabular form.

Part.	Area A.	h	A h^2	I_{CG}	$I_{CG}+A\,h^2$
(1)	25	5·81	844	13	857
(2)	37·6	·815	25	362·5	387·5
(3)	17·5	7·065	874	4·5	878·5

Σ 80·1 in.² Total I_{xx} = **2123 in.⁴**

$$\therefore \quad k^2 = \frac{I}{A} = \frac{2123}{80{\cdot}1} = 26{\cdot}5 \text{ in.}^2$$

The load W_e tends to open the gap in the frame. Hence max. fibre stress is tensile on edge nearest load and compressive on edge remote from load.

$$\text{Max. tensile stress } f_1 = \frac{W_e}{A}\left(1 + \frac{xy_1}{k^2}\right)$$

$$= \frac{39}{80{\cdot}1}\left(1 + \frac{17{\cdot}06 \times 7{\cdot}06}{26{\cdot}5}\right)$$

$$= {\cdot}487\ (1 + 4{\cdot}55)$$

$$= {\cdot}487 \times 5{\cdot}55 = \mathbf{2.70\ tons/in.^2}\ *$$

$$\text{Max. compress. stress } f_2 = \frac{W_e}{A}\left(1 - \frac{xy_2}{k^2}\right)$$

$$= {\cdot}487\left(1 - \frac{17{\cdot}06 \times 7{\cdot}94}{26{\cdot}5}\right)$$

$$= {\cdot}487 \times -4{\cdot}10 = -\mathbf{2{\cdot}0}\text{ tons/in.}^2$$

Note.—If direction of the load was reversed, f_1 would become compressive and f_2 tensile.

SECTION VI.

FLAT PLATES.

Preliminary Note.—The method used in this section for determining suitable dimensions for a flat plate, deals only with the most common type met in practice, namely, the bolted plate. For a more comprehensive study of flat plates, the student is referred to Mr. C. C. Pounder's pamphlet, "The Design of Flat Plates"

* The above tensile stress is too high. See Table 2, page 68, for allowable working stresses for cast iron.

(now out of print), which deals with special types, including ribbed and encastré plates, The following method gives satisfactory results in normal cases. The stress assumed, it should be noted, is the *average* skin stress, the maximum stress which occurs at the centre of the plate being about 1·25 times the average.

Example 16.—A rectangular opening 16″ × 10″ in a steam chest is to be closed by means of a C.I. flat plate cover (Fig. 14). The steam pressure is 80 lb./in.^{2} and the joint ($\frac{3}{4}$″ broad) may be assumed to be compressed to this pressure also. Design the flange and cover assuming the average stress in cast iron as 2500 lb./in.^{2} and 6000 lb./in.^{2} in the studs.

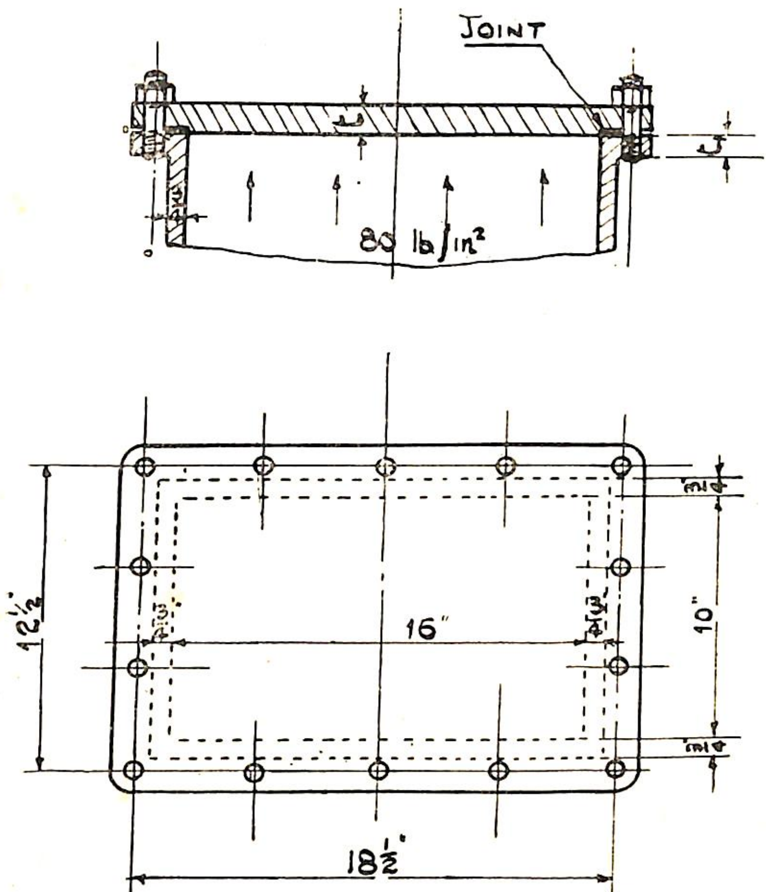


Fig. 14.

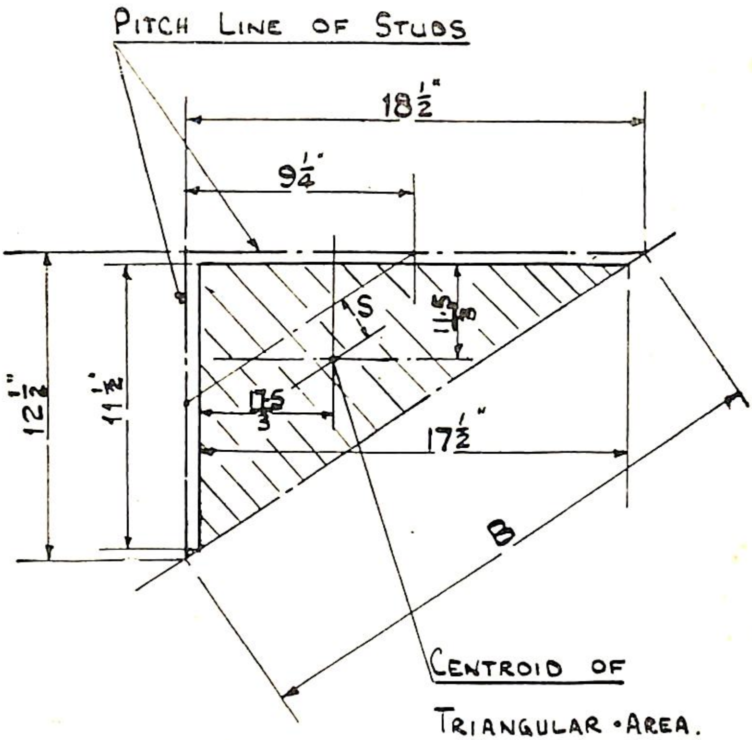


Fig. 15.

	Pressure on joint	= steam pressure.
∴	Total load on studs	= $(17\frac{1}{2}'' \times 11\frac{1}{2}'') \times 80$
		= 16100 lbs.
	Assume dia. of studs	= $\frac{5}{8}''$
	Area at bottom of thread	= $\cdot 204$ in.^{2}
Hence,	Load per stud	= $6000 \times \cdot 204 = 1225$ lbs.
∴	Number of studs	= $\dfrac{16,100}{1225}$ = about 13 studs.

Say 14 $\frac{5}{8}''$ studs.

Say centres of studs are $\frac{1}{2}''$ outside the packing,

∴	Pitch line of studs	= $(17\frac{1}{2}'' + 1'')$ by $(11\frac{1}{2}'' + 1'')$
		= $18\frac{1}{2}''$ by $12\frac{1}{2}''$

The studs are then arranged, as shown on Fig. 14.

The dimensions of the flange would be about $20\frac{1}{4}'' \times 14\frac{1}{4}''$

Cover—to find thickness t.

The simplest method is to take moments about the diagonal as illustrated in Fig. 15. The steam pressure is assumed to act at the centroid of the shaded triangular area, and to be resisted by the studs at the centroid of the pitch line. The distance S is the bending arm.

$$S = 1\tfrac{7}{8}'' \text{ by setting out to scale,}$$

$$\text{Total pressure on triangular area} = \frac{16{,}100}{2} = 8050 \text{ lbs.}$$

$$\text{Diagonal } B = \sqrt{18{\cdot}5^2 + 12{\cdot}5^2} = 22{\cdot}3''$$

$$\text{Bending Moment} = 8050 \times 1\tfrac{7}{8}'' = 15{,}100 \text{ in./lbs.}$$

$$\text{B.M.} = f\,Z$$

$$\therefore \quad 15{,}100 = 2500 \times \frac{Bt^2}{6}$$

$$\therefore \quad t^2 = \frac{15{,}100 \times 6}{22{\cdot}3 \times 2500} = 1{\cdot}62$$

$$\therefore \quad t = \sqrt{1{\cdot}62} = 1{\cdot}27'' \qquad \textbf{say } \mathbf{1\tfrac{5}{16}''}$$

Flange—to find thickness t_1,

Taking load on one stud producing bending on a section of flange of width one pitch.

$$\text{Min. pitch} = \frac{12{\cdot}5}{3} = 4{\cdot}166'' = b \text{ say.}$$

$$\text{Load on section} = \frac{16{,}100}{14} = 1150 \text{ lbs.}$$

$$\text{Bending arm} = \tfrac{1}{2}''$$

$$\therefore \quad \text{B.M.} = 1150 \times \tfrac{1}{2} = \mathbf{575 \text{ lbs.}}$$

$$\text{B.M.} = f\,Z \quad \frac{bd^2}{6} \times f$$

$$\therefore \quad 575 = \frac{4{\cdot}166 \times t_1^2}{6} \times 2500$$

$$t^2 = \frac{575 \times 6}{4{\cdot}166 \times 2500} = {\cdot}33$$

$$\therefore \quad t_1 = {\cdot}575''$$

To obtain sufficient depth for thread of studs, this would be increased to about $\tfrac{7}{8}''$

Example 17.—An 8″ dia. steel tube, $\frac{3}{8}$″ thick, has the ends closed by a flat steel cap which is held in place by a central bolt and cap nut, as shown in Fig. 16. The pressure in the tube is 200 lb./in.^{2} and the joint pressure may be taken as twice this. Find the bolt dia. and cap thickness allowing a safe stress of 9000 lb./in.^{2}
Neglecting bolt diameter,

Load upon opening $= 200 \times$ area 8″ dia.
$= 10{,}050$ lbs.

Load due to joint pressure $= 2 \times 200 \times (\pi \times 8\frac{3}{8}'' \times \frac{3}{8}'')$
$= 3940$ lbs.

$\therefore$ Total bolt load $= 10{,}050 + 3940 = 13{,}990$ lbs.
Stress $= 9000$ lb./in.^{2}

$\therefore$ Area at bottom of thread of bolt $= \dfrac{13{,}990}{9000}$
$= 1{\cdot}555$ in.^{2}

Nearest equivalent size of bolt above this area is $1\frac{3}{4}$″
Hence bolt dia. $=$ **$1\frac{3}{4}$″**

Cap Thickness t

Take moments about the effective diameter ($8\frac{3}{4}$″) of the cap. The total load tending to bend the cap is the sum of half the load on 8″ dia. opening plus half the joint load, and acts at the centroids in each case. These give a single resultant force which is opposed by the central bolt, this producing a couple balanced by the stress couple across the section of the cap.

Total bolt load for semi-circle $= \dfrac{13{,}990}{2} = 6995$ lbs.

Load on 8″ semi-circular area $= \dfrac{10{,}050}{2} = 5025$ lbs.

Load on joint for semi-circle $= \dfrac{3940}{2} = 1970$ lbs.

Fig. 17 illustrates the conditions of loading.

For a semi-circular area, centroid $= {\cdot}424 \times$ radius.
For a semi-circle, centroid $= {\cdot}636 \times$ radius.

Centroid of 8″ semi-circular area $= {\cdot}424 \times 4 = 1{\cdot}696''$
Centroid of joint area $= {\cdot}636 \times 4\frac{3}{16}'' = 2{\cdot}66''$

To find position of resultant, take moments about XX (see Fig. 17).

$\therefore$ $5025 \times 1{\cdot}696 + 1970 \times 2{\cdot}66 = 6995 \times S$
$8520 + 5250 = 6995 \times S$

$$\therefore \quad S = \frac{13{,}770}{6995} = \mathbf{1{\cdot}97''}$$

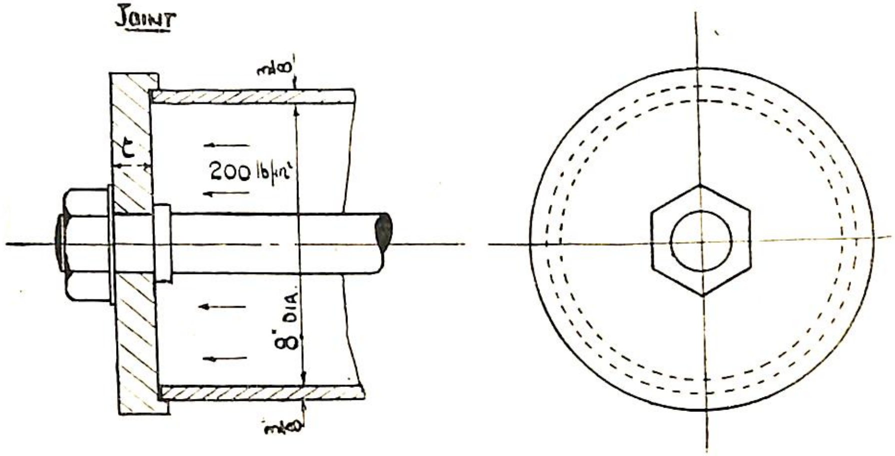

JOINT
t
200 lb/in²
8" DIA.

Fig. 16

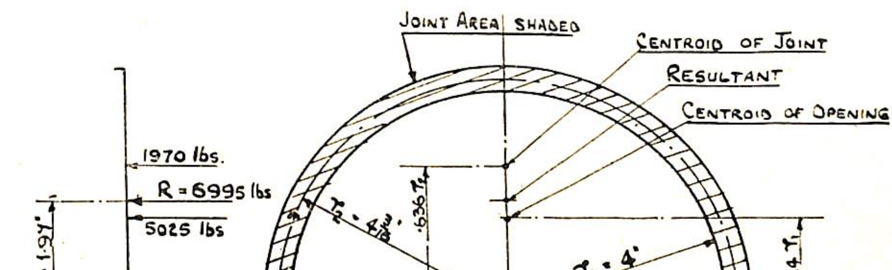

JOINT AREA SHADED
CENTROID OF JOINT
RESULTANT
CENTROID OF OPENING
1970 lbs.
R = 6995 lbs
5025 lbs
S = 1.97"
X
r₂ = 4 3/16"
.636 r₂
r₁ = 4"
.424 r₁
X

Fig. 17.

$\therefore$ Bending moment at XX $= 6995 \times 1{\cdot}97 = 13{,}770$ in. lbs.
Net width at XX $= 8\frac{3}{4}'' - 1\frac{3}{4}'' = 7''$

$\therefore$ B.M. $= fZ$

$$13{,}770 = 9000 \times \frac{7 \times t^2}{6}$$

$$t^2 = \frac{13{,}770 \times 6}{7 \times 9000} = 1{\cdot}31$$

$$\therefore \quad t = 1{\cdot}145 \qquad \text{say } 1\tfrac{3}{16}'' \text{ or } 1\tfrac{1}{4}''$$

Section VII.

THICK CYLINDERS

(Including Shrink and Force Fits).

General Formula.

$$t = \frac{d}{2}\left\{\sqrt{\frac{f+p}{f-p}} - 1\right\} \qquad \text{Lamé (1)}$$

where t = thickness of metal (ins.)
d = internal dia. of cylinder (ins.).
f = max. hoop stress (lb./in.^{2})
p = radial pressure (lb./in.^{2}).

For Internal Pressures.

$$f_{a1} = p\,\frac{b^2 + a^2}{b^2 - a^2} \text{ (max. stress)} \qquad (2)$$

$$f_{b1} = \frac{2\,pa^2}{b^2 - a^2} \qquad (3)$$

For External Pressures.

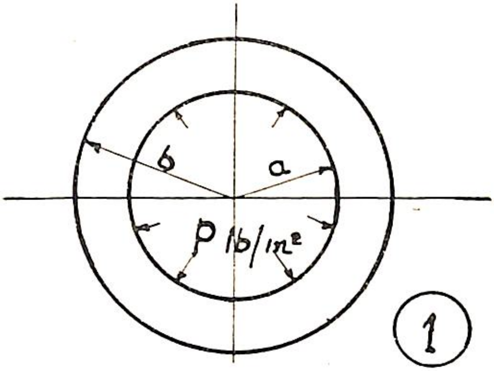
$$f_{a2} = -\frac{2\,pb^2}{b^2 - a^2} \text{ (max. stress)} \qquad (4)$$

$$f_{b2} = -p\,\frac{b^2 + a^2}{b^2 - a^2} \qquad (5)$$

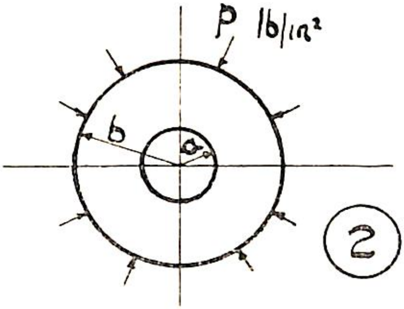
f_{a1} and f_{a2} = hoop stresses at radius a.
f_{b1} and f_{b2} = ,, ,, ,, b.

Other symbols as in accompanying sketch.

The maximum hoop stresses f_{a1} and f_{a2} correspond to the stress f in the general formula.



Internal Pressure.



External Pressure.

Sec. VII.

In expressions (4) and (5) the negative sign denotes compressive stress.

Expressions (2), (3), (4) and (5) are evolved from Lamé's general thick cylinder theory. These are useful in calculating the stresses in a shrink or force fit, since this same theory applies. An example is given later.

General Formula for Shrink or Force Fits.

$$\frac{S}{2} = \frac{R}{E} \quad (f_{a1} - f_{b2})$$

where f_{a1} and f_{b2} = hoop stresses at common radius of pieces.
R = common radius.
E = Young's modulus for material.
S = Total shrink allowance.

When the two pieces are of different material, the formula becomes

$$\frac{S}{2} = \frac{R}{E_1} (f_{a1} - \sigma_1 p) - \frac{R}{E_2} (f_{b2} - \sigma_2 p)$$

E_1 and E_2 = Modulus of elasticity for different materials.
σ_1 and σ_2 = Poisson's ratio for different materials.
Other symbols as already stated.
Note that p is negative in this formula.

Example 18.—A cast steel cylinder for a hydraulic press has an inside dia. of 9″. Water pressure = 2 tons/in.². Taking the maximum hoop stress as 10,000 lb./in.², find the thickness of metal required by Lamé formula.

$$t = \frac{d}{2}\left\{\sqrt{\frac{f+p}{f-p}} - 1\right\} \qquad (1)$$

$$= \frac{9}{2}\left\{\sqrt{\frac{10{,}000 + 2 \times 2240}{10{,}000 - 2 \times 2240}} - 1\right\}$$

$$= 4{\cdot}5\left\{\sqrt{\frac{14{,}480}{5520}} - 1\right\}$$

$$= 4{\cdot}5\,(\sqrt{2{\cdot}62} - 1)$$
$$= 4{\cdot}5\,(1{\cdot}62 - 1)$$
$$= 4{\cdot}5 \times {\cdot}62 = 2{\cdot}79'' \quad \textbf{say } 2\tfrac{7}{8}''$$

Example 19.—A cast iron hydraulic cylinder has an internal dia. = 6″ and external dia. = 8″. Water pressure = 750 lb./in.². What is the max. hoop stress? Also find the stress at the outer surface.

The max. hoop stress occurs at the internal diam.

Max. Stress.

$$f_{a1} = p \, \frac{b^2 + a^2}{b^2 - a^2} \qquad (2)$$

$$= 750 \times \frac{4^2 + 3^2}{4^2 - 3^2}$$

$$= 750 \times \frac{16 + 9}{16 - 9}$$

$$= 750 \times \frac{25}{7} = \mathbf{2680\ lb./in.^2}$$

Stress at Outer Surface.

$$f_{b1} = \frac{2\,p\,a^2}{b^2 - a^2} \qquad (3)$$

$$= \frac{2 \times 750 \times 3^2}{4^2 - 3^2}$$

$$= \frac{2 \times 750 \times 9}{7}$$

$$= 750 \times \frac{18}{7} = \mathbf{1930\ lb./in.^2}$$

These stresses are tensile, and hence for cast iron are about the limiting values.

Example 20.—A M.S. universal coupling end is shrunk on to a M.S. shaft, 8″ dia. (Fig. 18). The shrink allowance is ·001″ per inch dia.

(*a*) Find the stresses in shaft and coupling end.
(*b*) For a max. torque of 250 in. tons, find the factor of safety between the actual torque and the theoretical torque capable of being transmitted.

(*a*) **Coupling End.**

$$a = 4''$$
$$b = 6\tfrac{3}{4}''$$

At inner radius,

$$f_{a1} = p\ \frac{b^2 + a^2}{b^2 - a^2} \qquad (2)$$

$$= p \times \frac{6{\cdot}75^2 + 4^2}{6{\cdot}75^2 - 4^2}$$

$$= p \times \frac{45{\cdot}56 + 16}{45{\cdot}56 - 16}$$

$$= \mathbf{2{\cdot}08}\ P$$

At outer radius,

$$f_{b1} = \frac{2\ p\ a^2}{b^2 - a^2} \qquad (3)$$

$$\frac{2 \times p \times 4^2}{6{\cdot}75^2 - 4^2}$$

$$= p \times \frac{32}{29{\cdot}56}$$

$$= \mathbf{1{\cdot}08}\ p$$

We have now the two hoop stresses in coupling end in terms of p the radial stress.

Consider Shaft.

For a solid shaft the hoop stress at outside radius is equal to the radial stress, since $a = o$.

$$i.e., \quad f_{b2} = -\,p$$

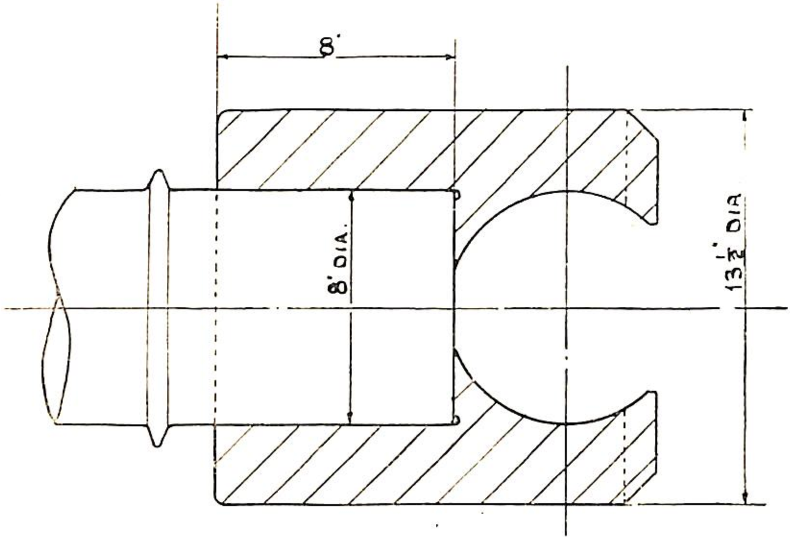


Fig. 18.

f_{b2} being used to correspond to the symbol in formula for a hollow shaft or cylinder externally loaded.

$$\frac{S}{2} = \frac{R}{E}(f_{a1} - f_{b2})$$

S total shrink allowance $= \cdot001'' \times 8 = \cdot008''$
R $= 4''$
E $= 30 \times 10^6$ lb./in.^{2} for steel.

Substituting f_{a1} and f_{b2} in terms of p

$$\text{Hence} \quad \frac{\cdot008}{2} = \frac{4}{30 \times 10^6}\left\{2\cdot08\,p - (-p)\right\}$$

$$\cdot004 = \frac{4}{30 \times 10^6} \times 3\cdot08\,p$$

$$p = \frac{\cdot004 \times 30 \times 10^6}{4 \times 3\cdot08}$$

$$= 9750 \text{ lb./in.}^2 \text{ (radial stress).}$$

$$\therefore \quad f_{a1} = 2\cdot08\,p = 2\cdot08 \times 9750$$
$$= 20{,}250 \text{ lb./in.}^2$$
$$f_{b1} = 1\cdot08\,p = 1\cdot08 \times 9750$$
$$= 10{,}500 \text{ lb./in.}^2$$

These are the maximum and minimum hoop stresses in the coupling end.

For shaft, $f_b = -p = -9750$ lb./in.^{2}

The permissible max. hoop stress f_{a_1} should be within the elastic limit for the material. In this case the above stress is about the maximum allowable.

(*b*) From previous evaluation,

Radial stress $p = 9750$ lb./in.^{2}
Assume coeff. of friction $= \cdot 25$
(This is a moderate estimate).

The radial stress p is the gripping force between the coupling end and the shaft

$$\text{Tangential force} = 9750 \times \cdot 25 = 2440 \text{ lb./in.}^2$$

$$\text{This acts on a surface area} = \pi \times D \times L = \pi \times 8'' \times 8'' = 201 \text{ in.}^2$$

$$\therefore \quad \text{Total tangential force} = \frac{2440 \times 201}{2240} = 219 \text{ tons.}$$

$$\text{Shaft radius} = 4''$$

$$\therefore \quad \text{Slip torque} = 219 \times 4 = 876 \text{ in. tons.}$$

This is the theoretical maximum torque capable of being transmitted without slip, due to the gripping force of the shrinkage.

Actual torque = 250 in. tons.

$$\therefore \quad \text{Factor of safety} = \frac{876}{250} = \mathbf{3 \cdot 5}$$

SECTION VIII.

FLYWHEELS.

Centrifugal Stress for a Rim.

$$f = \frac{12\, w\, v^2}{g}$$

where f = centrifugal stress (lb./in.^{2})
w = weight of material (lb./in.^{3})
v = velocity at mean radius (ft./sec.).
g = 32·2 ft./sec.^{2}

Energy Formulae.

$$\text{K.E.} = \tfrac{1}{2}\, I\, \omega^2 = \frac{W\, v^2}{2\, g}$$

$$I = \frac{Wk^2}{g}$$

$$\triangle_e = I\,\omega^2 q$$

$$T = \frac{I\,\omega}{t}$$

$$k = \sqrt{\frac{R_1^2 + R_2^2}{2}}$$

$$\omega = \frac{2\,\pi\,N}{60} = \frac{v}{r}$$

$$v = \frac{\pi\,D\,N}{60}$$

Notation.

K.E. = kinetic energy of flywheel (ft. lbs.).
I = inertia of flywheel (ft. lbs. sec.^{2}).
W = weight of rim (lbs.).
ω = angular velocity (rad./sec.).
v = linear velocity (ft./sec.).
k = radius of gyration (ft.).
g = 32·2 ft./sec.^{2}
$\triangle_e$ = surplus or increase in K.E. (ft. lbs.).
q = coeff. of speed fluctuation

$$= \frac{\text{Max. speed} - \text{min. speed}}{\text{mean speed.}}$$

T = average torque applied (ft. lbs.).
t = time to get up speed from rest (secs.).
R_1 = outer radius of rim (ft.).
R_2 = inner radius of rim (ft.).
N = revs. per min.
D = mean dia. of rim (ft.).

Note.—$\triangle_e$ is the energy given up by the flywheel in doing useful work, and is the difference between the maximum and minimum kinetic energy through a definite speed fluctuation q.

Suitable values of q for various machines are given in Table 5 in the Appendix.

Example 21.—A gas engine develops 12 I.H.P. at 350 R.P.M. and works on the 4-stroke cycle. Find a suitable size of flywheel if the energy absorbed is to be ·8 of the energy developed per cycle. Take speed fluctuation as ·5% up and down. The average rim stress should not exceed 1000 lb./in.^{2}

For cast iron, weight $= \cdot 26$ lb./in.^{3}

$$\text{Work done per rev.} = \frac{\text{H.P.} \times 33{,}000}{\text{Working cycles/min.}}$$

$$= \frac{12 \times 33{,}000}{350/2}$$

$$= 2265 \text{ ft. lbs.}$$

$\therefore$ Energy to be stored in flywheel $= \cdot 8 \times 2265$
or $\triangle_e = 1810$ ft. lbs.

To find permissible rim velocity for $f = 1000$ lb./in.^{2}

$$f = \frac{12\, w\, v^2}{g}$$

$$\therefore \quad 1000 = \frac{12 \times \cdot 26 \times v^2}{32 \cdot 2}$$

$$v^2 = \frac{1000 \times 32 \cdot 2}{12 \times \cdot 26} = 10{,}320$$

$$\therefore \quad v = 101 \cdot 6 \text{ ft./sec.} = \frac{\pi D N}{60}$$

$$\therefore \quad D = \frac{101 \cdot 6 \times 60}{\pi \times 350} = 5 \cdot 546 \text{ ft.}$$

Now $\triangle_e = I\, \omega^2 q$

$$q = \cdot 5\% \text{ up and } \cdot 5\% \text{ down} = 1\% = \frac{1}{100}$$

$$\omega = \frac{2 \pi N}{60} = \frac{2 \pi \times 350}{60} = 36 \cdot 7 \text{ rad./sec.}$$

$$\therefore \quad 1810 = I \times 36 \cdot 7^2 \times \frac{1}{100}$$

$$\therefore \quad I = \frac{181{,}000}{36 \cdot 7^2} = 134 \cdot 5 \text{ ft. lbs. sec}^2$$

$$\text{Now} \quad I = \frac{W k^2}{g}$$

$$\text{and } k = \frac{5 \cdot 546}{2} = 2 \cdot 773 \text{ ft.}$$

$$\therefore \quad 134 \cdot 5 = \frac{W \times 2 \cdot 773^2}{32 \cdot 2}$$

$$\therefore \quad W = \frac{134 \cdot 5 \times 32 \cdot 2}{2 \cdot 773^2} = \mathbf{564\ lbs.}$$

$$\therefore \quad \text{Vol. of rim} = \frac{564}{\cdot 26} = 2165 \text{ in.}^3$$

$$\text{Mean circumference} = \pi \times 5 \cdot 546 \times 12 = 209 \text{ ins.}$$

$$\therefore \quad \text{Area of rim} = \frac{2165}{209} = 10 \cdot 38 \text{ in.}^2$$

$$\begin{aligned} \text{Area} &= b \times t \text{ (breadth} \times \text{thickness)} \\ \text{say } b &= 1 \cdot 2\, t \\ \therefore \quad 10 \cdot 38 &= 1 \cdot 2\, t \times t \\ t^2 &= \frac{10 \cdot 38}{1 \cdot 2} = 8 \cdot 64 \\ t &= 2 \cdot 94'' \quad \text{say } 3'' \\ \therefore \quad b &= \frac{10 \cdot 38}{3} = 3 \cdot 45 \quad \textbf{say } \mathbf{3\tfrac{1}{2}''} \end{aligned}$$

Example 22.—A motor generator set for a rolling mill has a cast steel flywheel designed to take up the fluctuation of demand. The motor and generator weigh 20 tons and have a radius of gyration of 18″. The flywheel weighs 50 tons and has a mean rim radius of 5′-3″. The set runs at 500 R.P.M. For this speed, calculate the average stress in the flywheel rim due to centrifugal force. Take weight of cast steel = ·28 lb./in.^{3}.

If when the mill is running, an excess of energy of 8×10^6 ft. lb. is required above that instantaneously supplied by the motor, calculate the decrease of speed of the set and the coeff. of speed fluctuation.

$$\begin{aligned} \text{Total} \quad I &= \frac{20 \times 2240}{32 \cdot 2} \times 1 \cdot 5^2 + \frac{50 \times 2240}{32 \cdot 2} \times 5 \cdot 25^2 \\ &= 3130 + 96{,}000 = 99{,}130 \text{ ft. lb. sec.}^2 \\ &= 9 \cdot 913 \times 10^4 \text{ ft. lb. sec.}^2 \end{aligned}$$

To find centrifugal stress,

$$f = \frac{12\, w\, v^2}{g}$$

$$v = \frac{\pi D N}{60} = \frac{\pi \times 10 \cdot 5 \times 500}{60} = 27 \cdot 5 \text{ ft./sec.}$$

$$\therefore \quad f = \frac{12 \times \cdot 28 \times 27 \cdot 5^2}{32 \cdot 2} = 7900 \text{ lb. in.}^2$$

Decrease of Speed.

$$\begin{aligned}
\text{Initial K.E.} &= \tfrac{1}{2}\, I\, \omega_1^2 \\
&= \tfrac{1}{2} \times 9{\cdot}913 \times 10^4 \times 52{\cdot}4^2 \\
&= 136 \times 10^6 \text{ ft. lbs.} \\
\text{Excess energy given up} &= 8 \times 10^6 \text{ ft. lbs.} \\
\therefore\ \text{Final K.E.} &= 136 \times 10^6 - 8 \times 10^6 \\
&= 128 \times 10^6 \text{ ft. lbs.} \\
&= \tfrac{1}{2}\, I\, \omega_2^2 \\
\therefore\ 128 \times 10^6 &= \tfrac{1}{2} \times 9{\cdot}913 \times 10^4 \times \omega_2^2 \\
\omega_2^2 &= \frac{128 \times 10^6 \times 2}{9{\cdot}913 \times 10^4} = 2580 \\
\therefore\ \omega_2 &= 50{\cdot}8 = \frac{2\,\pi\,N_2}{60} \\
\therefore\ N_2 &= \frac{50{\cdot}8 \times 60}{2\,\pi} = 485
\end{aligned}$$

$\therefore$ **Decrease** in speed $= 500 - 485 =$ **15** R.P.M.

The speed fluctuation

$$q = \frac{\text{Max. speed} - \text{Min. speed}}{\text{Mean Speed}}$$

$$= \frac{500 - 485}{492{\cdot}5} = {\cdot}03$$

Example 23.—A flywheel weighs 2 tons and has a radius of gyration of 4′-0″. It is keyed to the crankshaft of an engine which develops 20 H.P. at 150 R.P.M. Assuming the mean torque constant at all speeds, and neglecting all masses but the flywheel, find the time required by the engine to get up full speed.

$$\text{Average torque } T = \frac{\text{H.P.} \times 63{,}000}{N}$$

$$= \frac{20 \times 63{,}000}{150} = 8400 \text{ in. lbs.}$$

$$= 700 \text{ ft. lbs.}$$

$$T = \frac{I\,\omega}{t}$$

$$I = \frac{W k^2}{g} = \frac{2 \times 2240 \times 4^2}{32{\cdot}2}$$

$$= 2230 \text{ ft. lbs. sec.}^2$$

$$\omega = \frac{2\,\pi\,N}{60} = \frac{2\,\pi \times 150}{60} = 15{\cdot}7 \text{ rad/sec.}$$

$$\therefore \qquad 700 = \frac{2230 \times 15{\cdot}7}{t}$$

$$t = \frac{2230 \times 15{\cdot}7}{700} = 50 \text{ secs.}$$

Example 24.—A C.I. flywheel for a shearing machine has a rim section $9\frac{1}{2}''$ square. The outside dia. is 67″ and the inside dia. 48″. The flywheel is belt-driven by an electric motor running at 600 R.P.M. The motor pulley is 19″ dia. The energy absorbed during the effective cutting stroke of 3″ is 80% of the energy supplied. When cutting a section, the speed of the flywheel is reduced by 15 R.P.M. Determine the average cutting force exerted at the blade. Taking the shearing strength of steel at 23 tons/in.², find the area of steel being sheared. The effect of the flywheel arms may be neglected. Weight of cast iron $= \cdot 26$ lb./in.³.

$$\text{Weight of rim} = (\text{area } 67'' - \text{area } 48'') \times 9{\cdot}5 \times {\cdot}26$$
$$= 1716 \times 9{\cdot}5 \times {\cdot}26 = \mathbf{4240 \ lbs.}$$

$$\text{Max. speed of flywheel} = \frac{600 \times 19}{67} = 170 \text{ R.P.M.} = N_1 \text{ say.}$$

$$\text{Rad. of gyration} \quad k = \sqrt{\frac{\left(\frac{67}{2}\right)^2 + \left(\frac{48}{2}\right)^2}{2}}$$

$$= \sqrt{849} = 29{\cdot}14'' = \mathbf{2{\cdot}425} \text{ ft.}$$

At 170 R.P.M.

$$\omega_1 = \frac{2\,\pi\,N_1}{60} = \frac{2\,\pi \times 170}{60} = 17{\cdot}8 \text{ rad. sec.}$$

Lower speed of flywheel $= 170 - 15 = 155$ R.P.M. $= N_2$ say

At 155 R.P.M.

$$\omega_2 = \frac{2\,\pi\,N_2}{60} = \frac{2\,\pi \times 155}{60} = 16{\cdot}25 \text{ rad./sec.}$$

$$I = \frac{W\,k^2}{g} = \frac{4240 \times 2{\cdot}425^2}{32{\cdot}2} = 774 \text{ ft. lb. sec.}^2$$

$$\therefore \quad \text{K.E. at 170 R.P.M.} = \tfrac{1}{2}\,I\,\omega_1^2$$
$$= \tfrac{1}{2} \times 774 \times 17{\cdot}8^2 = 122{,}500 \text{ ft. lbs.}$$

$$\text{K.E. at 155 R.P.M.} = \tfrac{1}{2}\,I\,\omega_2^2$$
$$= \tfrac{1}{2} \times 774 \times 16{\cdot}25^2 = 102{,}000 \text{ ft. lbs.}$$

$\therefore$ Energy given out $= 122{,}500 - 103{,}000 = 20{,}500$ ft. lbs.
Energy absorbed or work done at blade $= 20{,}500 \times \cdot 8$
$= 16{,}400$ ft. lbs.

Also work done = average cutting force $\times \dfrac{3}{12}$ ft.

$$\therefore \text{ Average cutting force} = \frac{12 \times 16{,}400}{3}$$

$$= 65{,}500 \text{ lbs.} = \mathbf{29{\cdot}2 \text{ tons.}}$$

$$\text{Area of section being sheared} = \frac{29{\cdot}2}{23} = \mathbf{1{\cdot}27 \text{ in.}^2}$$

SECTION IX.

GEARING.

To cover the design of all types of gearing is scarcely possible in the present pamphlet. It is the intention, therefore, to include here only two examples of the primary form of tooth gearing—one example dealing with spur gearing and the other with bevel gearing. As far as possible, the vital features connected with the design of these two types will be dealt with, although the student is recommended to reinforce the material given here with general reading from a good text-book.

(*a*) Spur Gearing.

Formulae.

$$E = \frac{T}{r} = \frac{63{,}000 \times \text{H.P.}}{r \times \text{R.P.M.}} = \frac{33{,}000 \times \text{H.P.}}{V}$$

Lewis formula.

$$E = y\, b\, P\, f_w$$

$$n = \frac{\pi \times \text{P.C.D.}}{P} = \text{P.C.D.} \times \text{D.P.}$$

$$\text{D.P.} = \frac{\pi}{P}$$

Notation.

E = Tangential load on teeth (lbs.).
T = Torque transmitted (in lbs.).
r = Pitch circle radius (ins.).
V = Pitch line velocity (ft./min.).
y = Lewis form factor (see Notes).
b = Width of teeth (ins.).
P = Circular pitch (ins.).
f_w = Safe working stress (lb./in.^{2}).
n = Number of teeth.
P.C.D. = Pitch circle diam. (ins.).
D.P. = Diametrical pitch.

Values of y.

y is a factor which depends on the shape and number of teeth. The following are the usual values :—

$$14\tfrac{1}{2}^\circ \text{ involute, full depth, } \quad y = \cdot 124 - \frac{\cdot 684}{n}$$

$$20^\circ \quad ,, \quad ,, \quad ,, \quad y = \cdot 154 - \frac{\cdot 912}{n}$$

$$20^\circ \quad ,, \quad \text{stub tooth, } \quad y = \cdot 175 = \frac{\cdot 90}{n}$$

Working Stress f_w

Allowable values for f_w are given in Table 6 in Appendix, these covering the more common materials used in gear manufacture. Allowance is made for the dynamic action at various pitch line velocities.

Width of Teeth.

In determining the proportions of wheel teeth by the Lewis formula, the usual practice is to fix the width in terms of the circular pitch, and substitute. The correct ratio to adopt depends on the class of work and the designer's experience.

Generally, for slow speeds and cases where shafts are inaccurately adjusted, the face width may be $1\frac{1}{4}$ to $2\frac{1}{2}$ times the pitch. For average work, a value of three to four times the pitch may be used. In the case of small high-speed gearing where smooth engagement is necessary, the face width may be six to eight times the pitch. This also applies to cases where wear is of importance. The high ratios, it should be noted, result in finer tooth pitches.

Wheel Arms.

Fig. 19 shows the more common sections used for wheel arms. Types (a) and (b) apply to small and medium-sized wheels, while type (d) is common for large wheels.

The arms are designed to resist the bending effect of the tooth load, the maximum bending occurring where the arm meets the wheel boss. It is usual to neglect the effect of the side ribs in types (b) and (c) and the centre rib in type (d), these being considered as stiffeners, and only the shaded parts (see Fig. 19) proportioned to take the load. The tooth load is assumed to be equally divided among the arms.

Example 25.—Design a pair of involute spur gears to transmit 25 H.P. at 500 R.P.M. Gear ratio, 4 to 1. Assume a 20-teeth pinion, and ratio of breadth to pitch = 4. Pinion material forged steel, and wheel cast steel.

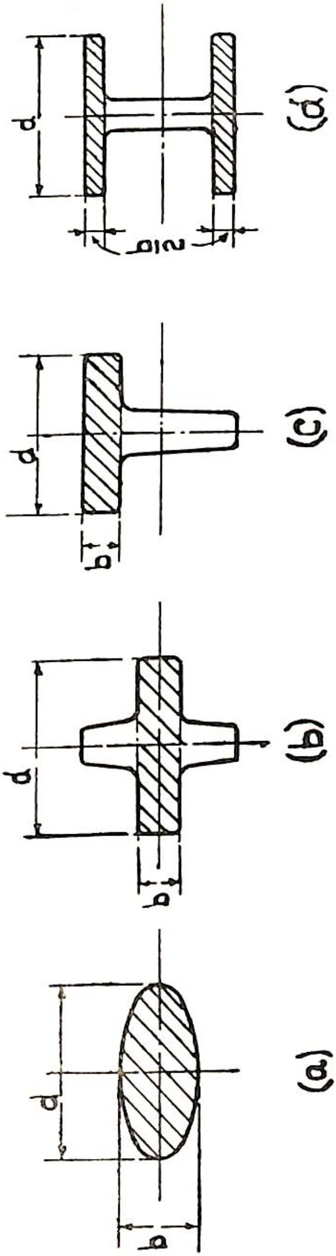


Fig. 19.

Method.—First determine tooth proportions of pinion and check for wheel.

$$T = \frac{\text{H.P.} \times 63{,}000}{\text{R.P.M.}}$$

$$= \frac{25 \times 63{,}000}{500} = 3150 \text{ in. lbs.}$$

i.e., $T = E \times r = 3150$ in. lbs.

or $E \times \dfrac{\text{P.C.D.}}{2} = 3150$

Now $\text{P.C.D.} = \dfrac{n\,P}{\pi}$

$\therefore \quad E \times \dfrac{n\,P}{2\,\pi} = 3150$

$$\therefore \quad E = y\,b\,P\,f_w = \frac{2\,\pi \times 3150}{n\,P} \qquad (1)$$

Assume teeth are $14\frac{1}{2}°$ involute.

$$\therefore \quad y = \cdot 124 - \frac{\cdot 684}{n} \quad (n = 20 \text{ teeth}).$$

$$= \cdot 124 - \frac{\cdot 684}{20}$$

$$= \cdot 124 - \cdot 034 = \cdot 090$$

Given $b = 4\,P$

To decide a suitable value for f_w, assume meantime P.C.D. of pinion $= 5''$

Hence pitch line velocity $v = \pi \times \dfrac{\text{P.C.D.}}{12} \times$ R.P.M.

$= \pi \times 5/12 \times 500$

$=$ about 650 ft./min.

From Table 6, for forged steel pinion,

$f_w = 12{,}000$ lb./in.^{2}

Substituting in equation (1)

$$\cdot 090 \times 4\,P \times P \times 12{,}000 = \frac{2\,\pi \times 3150}{20 \times P}$$

$$\therefore \quad P^3 = \frac{2\,\pi \times 3150}{\cdot 090 \times 4 \times 12{,}000 \times 20}$$

$$= \cdot 229$$

$$\therefore \quad P = \cdot 612''$$

$$\therefore \quad \text{D.P.} = \frac{\pi}{\text{P}} = \frac{\pi}{\cdot 612} = 5 \text{ D.P. (nearest).}$$

$\therefore$ Pitch dia. of pinion $= 20/5 = 4''$

$\therefore$ Actual circular pitch $= \pi/5 = \cdot 628''$

This is slightly coarser than calculated value, and gives a stronger tooth, hence 5 D.P. is correct.

Breadth of teeth $= 4\text{ P} = 4 \times \cdot 628 = 2\tfrac{1}{2}''$

$$\text{Actual } f_w = \frac{\text{E}}{y\, b\, \text{P}}$$

$$= \frac{1575}{\cdot 090 \times 2\cdot 5 \times \cdot 628} = 11{,}150 \text{ lb./in.}^2$$

$$\text{Actual pitch line velocity} = \pi \times \frac{4}{12} \times 500 = 523 \text{ ft./min.}$$

As this is less than the assumed velocity, the above stress is safe, and values need not be recapitulated.

Number of teeth in pinion $= 20$

Gear ratio, 4 to 1.

$\therefore$ Number of teeth in wheel $= 20 \times 4 = 80$

For turning the gears before cutting teeth, the blank diameters are required.

$$\text{Blank or Outside dia.} = \frac{n + 2}{\text{D.P.}}$$

$$\text{For Pinion, O. dia.} = \frac{20 + 2}{5} = 22/5 = 4\cdot 40''$$

$$\text{For wheel, O. dia.} = \frac{80 + 2}{5} = 82/5 = 16\cdot 40''$$

Tangential load at pitch line,

$\text{E} = 3150/2 = 1575$ lbs.

$$\text{Load per inch of tooth width} = \frac{1575}{2\cdot 5} = 630 \text{ lbs.}$$

Summarised Particulars.

ITEM.	No. of Teeth.	D.P.	P.C. Dia.	Teeth Width.	Blank Dia.
F.S. Pinion,	20	5	4″	2½″	4·4″
C.S. Wheel,	80	5	16″	2½″	16·4″

Check on Stress in Wheel Teeth.

$$f_w = \frac{E}{y\,b\,P}$$

For 80 teeth, $y = \cdot124 - \dfrac{\cdot684}{80}$

$= \cdot124 - \cdot00855$
$= \cdot1154$

$\therefore$ For wheel, $f_w = \dfrac{1575}{\cdot1154 \times 2\cdot5 \times \cdot628} = 8680$ lb./in.^{2}

Pitch line velocity = 523 ft./min.
From Table 6, at 600 ft./min. velocity,
f_w for cast steel = 10,000 lb./in.^{2}
Wheel teeth are within safe limits.

Wheel and Pinion Shafts.

These would be designed as shown in Example 10. Section III. and calculation need not be repeated. The wheel shaft works out at $2\frac{1}{4}''$.

Wheel Arms.

Since the pinion is small, it would be forged solid and bored out for the shaft. The wheel, however, would have arms, and for a wheel of this size, four arms would be sufficient, of section either (a) or (b), Fig. 19.

Suppose the arms are as shown at (a), *i.e.*, elliptical.

Say wheel boss dia. $= 4\frac{1}{2}''$
$\therefore$ Bending arm $= 8'' - 2\frac{1}{4}'' = 5\frac{3}{4}''$
Tangential tooth load = 1575 lbs.
Load per wheel arm = 1575/4 = $393\frac{3}{4}$ lbs.
$\therefore$ Bending moment $= 393\frac{3}{4} \times 5\frac{3}{4} = 2265$ in. lbs.
B.M. $= f\,Z$

For elliptical section, $Z = \pi/32\; b\, d^2$ [b = width of section. d = depth in direction of load.]

Say $b = \frac{3}{4}''$ and $f = 6000$ lb./in.^{2}

$$\therefore \quad 2265 = 6000 \times \frac{\pi}{32} \times \cdot75 \times d^2$$

$$\therefore \quad d^2 = \frac{2265 \times 32}{6000 \times \pi \times \cdot75} = 5\cdot12$$

$\therefore$ $d = 2\cdot265''$ say $2\frac{3}{8}''$

i.e., Arm section at root would be $2\frac{3}{8}'' \times \frac{3}{4}''$, which could taper to $2'' \times \frac{5}{8}''$ at the wheel rim.

Other Details.

The remainder of the design may be left to the student's judgment. Fig. 20 shows the final detail of the gears under consideration.

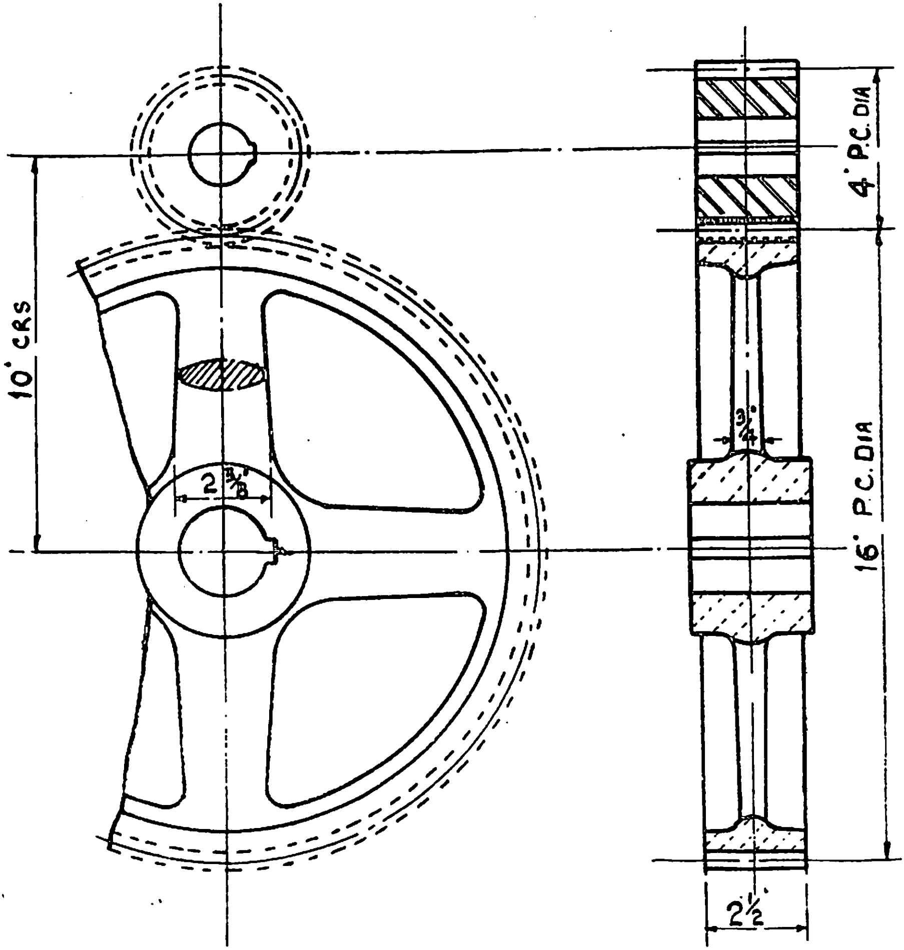


Fig. 20.—Spur Wheel and Pinion (Example 25)

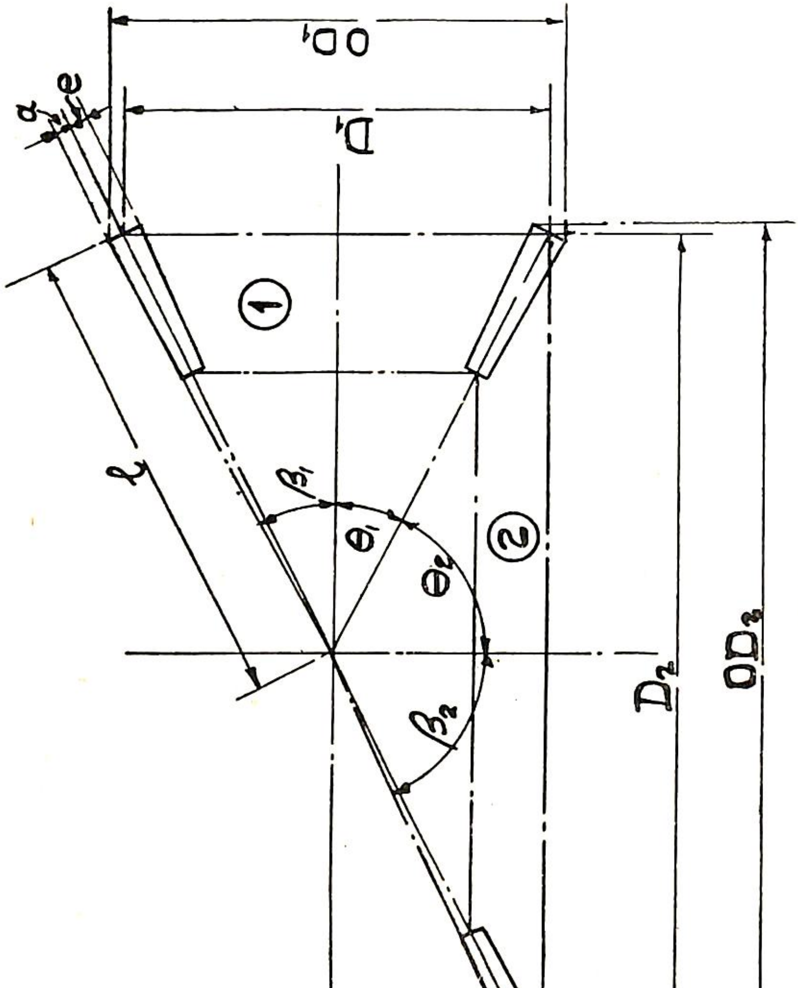


Fig. 21.

BEVEL GEARING.

Symbol.	Definition.	Formula.
n_1 n_2	No. of teeth in pinion, No. of teeth in wheel,	$n = \text{D.P.} \times D = \frac{\pi D}{P}$
D	Pitch Circle Diam., ...	$D = \frac{n P}{\pi} = \frac{n}{\text{D.P.}}$
O D	Outside or Blank Diam., ...	$OD = D + 2a \cos\theta$
a	Addendum, ...	$a = \frac{1}{\text{D.P.}} = \cdot 3183\ P$
e	Dedendum, ...	$e = \frac{1 \cdot 157}{\text{D.P.}} = \cdot 3683\ P$
θ_1	Pinion Pitch Cone Angle, ...	$\theta_1 = \tan^{-1} \frac{n_1}{n_2}$
θ_2	Wheel Pitch Cone Angle, ...	$\theta_2 = \tan^{-1} \frac{n_2}{n_1}$
β	Face Angle, ...	$\beta = \theta + \tan^{-1} \frac{a}{l}$
P	Circular Pitch, ...	$P = \frac{\pi D}{n} = \frac{\pi}{\text{D.P.}}$
D.P.	Diametrical Pitch, ...	$\text{D.P.} = \frac{n}{D} = \frac{\pi}{P}$
n^1	No. of teeth in Equivalent Spur Wheel.	$n^1 = \frac{n}{\cos\theta}$
l	Length of Pitch Cone Surface,	$l = \frac{D}{2 \sin\theta}$

Note.—Where the formula is common to both pinion and wheel, the suffix is omitted.

(*b*) BEVEL GEARING.

There are various specialised forms of bevel gears, viz., internal crown, etc. The most common type is used for driving shafts with their axes at right angles, and only this type will be dealt with in this section.

Fig. 21 shows a skeleton layout of a pair of bevel gears, the table below giving the more important formulae used in calculating the particulars required for the manufacture of a pair of gears.

Design of Wheel Teeth.

Since the bevel tooth is only a full tooth at the pitch cone diameter, the method for calculating the tooth proportions differs slightly from that used for ordinary spur wheels.

The following modified form of the Lewis formula gives satisfactory results. The tooth load is assumed to act at the mean radius of the tooth width and the formula is corrected to allow for this.

Modified Lewis Formula for Bevel Gears.

$$E = r^2 \, y \, b \, P \, f_w$$

Notation.

r = Ratio of mean pitch radius to pitch radius.
y = Form factor based on number of teeth in equivalent spur wheel.
E = Tangential load at pitch circle (lbs.).
b = Width of teeth (ins.).
P = Circular pitch (ins.).
f_w = Safe working stress (lb./in.^{2}).

Wheel Arms.

These are designed in a similar manner to spur wheel arms. The usual section is shown on Fig. 19, type (c).

Width of Teeth.

The teeth width usually varies from $\frac{1}{4}$ to $\frac{1}{3}$ the length of the pitch cone surface.

Example 26.—Design a pair of bevel gears to transmit 32 H.P. at 500 R.P.M. of driving shaft. Reduction ratio $2\frac{1}{2}$ to 1, and shafts at right angles. Both gears to be cast steel. Assume pinion has 24 teeth and involute angle = $14\frac{1}{2}°$.

$$\text{T.M.} = \frac{\text{H.P.} \times 63{,}000}{\text{R.P.M.}} = \frac{32 \times 63{,}000}{500} = 4030 \text{ in. lbs.}$$

Pitch Cone Angles.

n_1 = number of teeth in pinion $= 24$

n_2 = number of teeth in wheel $= 24 \times 2{\cdot}5 = 60$

$\therefore$ Pinion pitch cone angle $\theta_1 = \tan^{-1} \dfrac{n_1}{n_2}$

$$= \tan^{-1} \frac{24}{60}$$

$$= \tan^{-1} \cdot 4$$

$$= 21^\circ\ 48'$$

Shafts at Right Angles.

$\therefore$ Wheel pitch cone angle $\theta_2 = 90^\circ - 21^\circ\ 48'$

$$= 68^\circ\ 12'$$

Design of Teeth.

$$E = r^2 y\, b\, Pf_w$$

To find Ratio r

Let width of teeth = $\frac{1}{3}$ height of pitch cone surface.

i.e., $b = 1/3\ l$ (see Fig. 22).

Let r_1 = pitch radius of pinion.

r_m = mean pitch radius of pinion.

$\therefore$ By similar triangles,

$$\frac{r_m}{r_1} = \frac{5}{6} = r$$

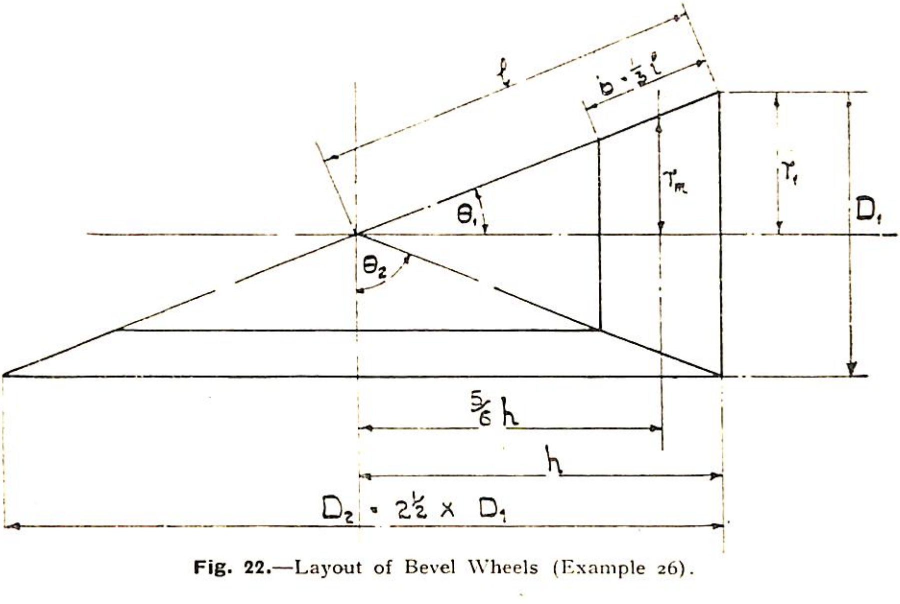


Fig. 22.—Layout of Bevel Wheels (Example 26).

To find b in terms of P.

$$l = \frac{D_1}{2 \sin \theta_1} = \frac{n_1 P}{2 \pi \sin \theta_1}$$

$$\therefore \quad b = \frac{n_1 P}{6 \pi \sin \theta_1} \qquad \begin{bmatrix} \theta_1 = 21^\circ 48' \\ n_1 = 24 \text{ teeth} \end{bmatrix}$$

$$= \frac{24 \times P}{6 \pi \times \cdot 3714}$$

$$= 3 \cdot 428 \text{ P}$$

To find y Value.

First find number of teeth in equivalent spur wheel.

$$\text{For pinion,} \quad n^1 = \frac{n_1}{\cos \theta_1}$$

$$= \frac{24}{\cdot 9285} = 25 \cdot 9, \qquad \text{say } 26.$$

$$\therefore \quad \text{For } 14\tfrac{1}{2}^\circ \text{ involute, } y = \cdot 124 - \frac{\cdot 684}{26}$$

$$= \cdot 124 - \cdot 024 = \cdot 100$$

$$\text{Now} \qquad \text{T.M.} = 4030 \text{ in. lbs.}$$

$$\text{Also} \qquad \text{T.M.} = \text{E} \times \frac{D_1}{2}$$

$$\text{or} \qquad 4030 = \text{E} \times \frac{n_1 P}{2 \pi}$$

$$\therefore \quad \text{E} = \frac{2 \pi \times 4030}{24 \times P} = \frac{336 \pi}{P}$$

Working Stress.

Assume meantime $f_w = 8000$ lb./in.^{2}

If this is too high for the actual pitch line velocity, the calculation will have to be revised when the pinion diameter is found.

$$\text{E} = r^2 \, y \, b \, \text{P} \, f_w$$

$$\therefore \quad \frac{336 \pi}{P} = \left(\frac{5}{6}\right)^2 \times \cdot 100 \times 3 \cdot 428 \text{ P} \times \text{P} \times 8000$$

$$\therefore \quad P^3 = \frac{336 \pi \times 36}{25 \times \cdot 100 \times 3 \cdot 428 \times 8000}$$

$$= \cdot 555$$

$$\therefore \quad P = \cdot 821 \text{ in.}$$

$$\therefore \quad \text{D.P.} = \frac{\pi}{\text{P}} = \frac{\pi}{\cdot 821} = 3{\cdot}83$$

To suit Standard Cutters, say 4 D.P.

$$\therefore \quad \text{Actual P} = \frac{\pi}{4} = \cdot 7854 \text{ in.}$$

$$\therefore \quad \text{For Pinion, pitch dia.,} \qquad D_1 = \frac{n_1}{\text{D.P.}} = \frac{24}{4} = 6''$$

$$\text{For Wheel, pitch dia.,} \qquad D_2 = \frac{n_2}{\text{D.P.}} = \frac{60}{4} = 15''$$

As the pinion is of same material as the wheel, the pinion teeth are weaker, hence check stress in pinion teeth only for actual pitch line velocity.

$$\text{Actual pitch line velocity} = \pi \times \frac{6}{12} \times 500$$
$$= 785 \text{ ft./min.}$$

$$\text{Actual} \quad f_w = \frac{E}{r^2\, y\, b\, P}$$

$$E = \frac{4030}{3} = 1343 \text{ lbs.} \quad \left[E = \frac{T}{r} \right]$$

$$b = 3{\cdot}428 \text{ P} = 3{\cdot}428 \times \cdot 7854 = 2{\cdot}693''$$

For practical purposes, this would be made $2\frac{3}{4}''$ without appreciably affecting the tooth strength.

$$\therefore \quad \text{Actual} \quad f_w = \frac{1343 \times 36}{25 \times \cdot 100 \times 2{\cdot}75 \times \cdot 7854}$$
$$= 8960 \text{ lb./in.}^2$$

From Table 6, it will be seen that this stress is within safe limits.

Blank or Outside Diameters.

For Pinion, $OD_1 = D_1 + 2a \cos \theta_1$

$$a = \frac{1}{\text{D.P.}} \qquad \theta_1 = 21° \, 48'$$

$$\therefore \quad OD_1 = 6 + 2 \times \tfrac{1}{4} \times \cdot 9285$$
$$= 6 + \cdot 464 \qquad = \mathbf{6{\cdot}464''}$$

For Wheel, $OD_2 = D_2 + 2a \cos \theta_2$

$$a = \frac{1}{\text{D.P.}}; \qquad \theta_2 = 68° \, 12'$$

$$OD_2 = 15 + 2 \times \tfrac{1}{4} \times \cdot 3714$$
$$= 15 + \cdot 186 \qquad = \mathbf{15{\cdot}186''}$$

Face Angles.

$$\beta = \theta + \tan^{-1}\frac{a}{l}$$

$$\text{For Pinion, } l = \frac{D_1}{2\sin\theta_1} = \frac{6}{2\sin 21°48'} = \frac{6}{2\times\cdot 3714} = \mathbf{8{\cdot}076''}$$

$$\therefore \quad \tan^{-1}\frac{a}{l} = \frac{1}{4\times 8{\cdot}076} = \cdot 0309$$

This is common to both pinion and wheel.

For Pinion.

$$\begin{aligned}\text{Face Angle } \beta_1 &= 21°\,48' + \tan^{-1}\cdot 0309\\ &= 21°\,48' + 1°\,47'\\ &= 23°\,35'\end{aligned}$$

For Wheel.

$$\begin{aligned}\text{Face Angle } \beta_2 &= 68°\,12' + 1°\,47'\\ &= 69°\,59'\end{aligned}$$

Summarised Particulars.

ITEM.	No. of Teeth.	D.P.	P.C. Dia	Width of Teeth.	Dia. of Blank.	Pitch Cone Angle.	Face Angle.
Pinion,	24	4	6″	2·75″	6·464″	21°48′	23°35′
Wheel,	60	4	15″	2·75″	15·186″	68°12′	69°59′

Wheel Arms.

As already stated, the wheel arms are designed in the same way as spur wheel arms, and the calculation for the above bevel gears need not be repeated. It should be noted, however, that the load on the arms is calculated from the torque at the mean pitch radius. The tangential tooth load at the centre of the teeth would be

$$E_1 = \frac{\text{T.M.}}{\text{Mean pitch radius}}$$

$$\text{and for above example, } E_1 = \frac{4030}{\frac{5}{6}\times 3} = \frac{4030}{2{\cdot}5} = \mathbf{1612\ lbs.}$$

The arms would then be designed to resist this load.

Other Details.—These can be left to the student's judgment. The final detail of these wheels is shown at Fig. 23.

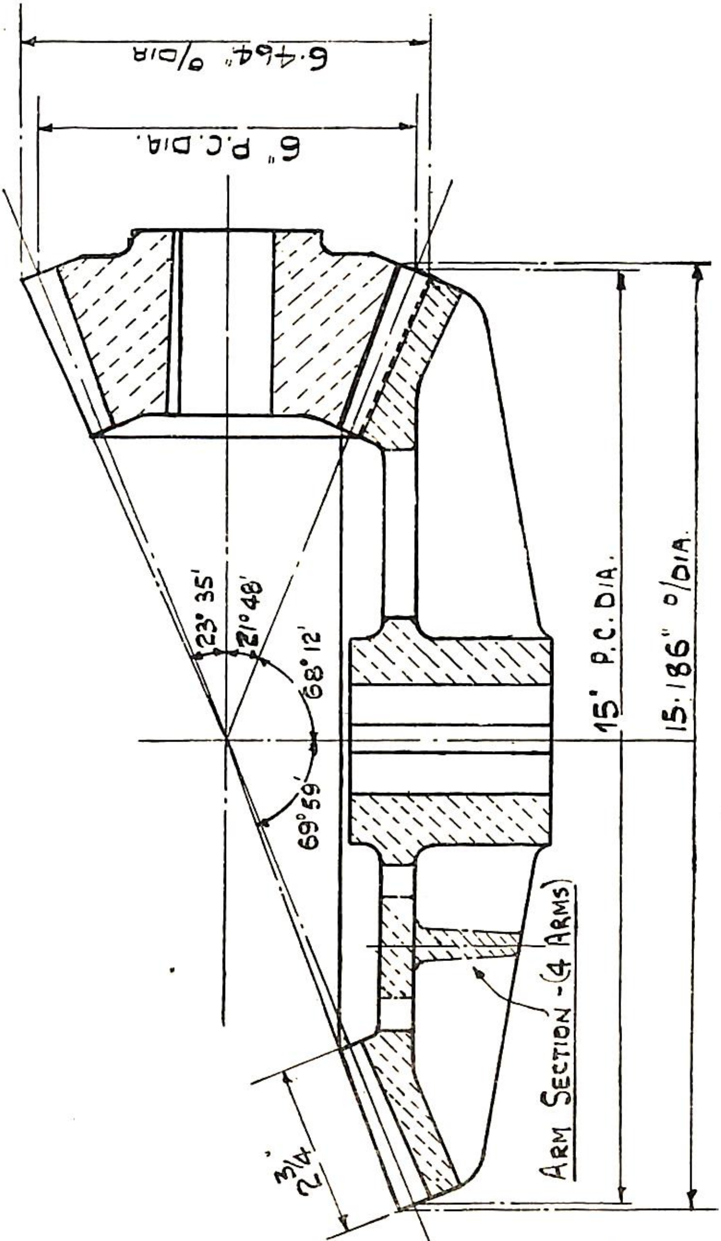


Fig. 23.—Bevel Gears (Example 26).

Concluding Notes on Gearing.

This section would not be complete without some reference to the standard gear reduction drives which are manufactured for all classes of power transmission. These gear units are totally enclosed with splash lubricated teeth, and high efficiencies are obtained by grinding the teeth. The usual spur reduction gear specification includes pinions made of case-hardened nickel chrome steel and wheels of high tensile forged or cast steel. Since each pinion tooth makes a greater number of contacts than each wheel tooth (depending on the reduction ratio) the above arrangement tends to give more equal wear on pinion and wheel teeth. This is a consideration in cases where length of service of the gears is an important factor.

APPENDIX.

TABLE I.

Ordinary Working Stresses (lb./in.^{2}). The Straining Action a steady or permanent one.

Material.	Kind of Stress.				
	Tension f_t	Compression f_c	Bending f_b	Shear f_s	Torsion.
Cast Iron ...	4,200	12,000	6,000 to 8,000	4,000	4,000 to 6,000
Mild Steel, ...	13,000 to 17,000	13,000 to 17,000	13,000 to 17,000	10,000 to 13,000	8,000 to 12,000
Cast Steel, ...	17,000 to 21,000	17,000 to 21,000	17,000 to 21,000	13,000 to 17,000	12,000 to 16,000
Steel Castings, ...	8,000 to 12,000	12,000 to 16,000	10,000 to 14,000	7,000 to 12,000	7,000 to 12,000
Phos. Bronze, ...	10,000	—	—	7,000	4,200
Gun-Metal, ...	4,200	—	—	—	—
Rolled Copper, ...	6,000	—	—	2,400	—
Brass, ...	3,000	—	—	—	—

TABLE 2.

Ordinary Working Stresses (lb./in.^{2}). Straining Action producing Stress of one kind only, varying from zero to a maximum value.

MATERIAL.	Tension f_t	Compression f_c	Bending f_b	Shear f_s	Torsion.
Cast Iron, ...	2,800	8,500	4,000	3,500	2,800
Mild Steel, ...	8,600 to 11,400	8,600 to 12,000	8,600 to 11,400	6,500 to 8,600	5,300 to 8,000
Cast Steel, ...	11,400 to 14,000	11,400 to 14,000	11,400 to 14,000	8,600 to 11,400	8,000 to 10,600
Steel Castings, ...	5,300 to 8,000	8,000 to 10,600	6,600 to 9,400	4,700 to 8,000	4,700 to 8,000
Phos. Bronze, ...	6,600	—	—	4,600	2,800
Gun-Metal, ...	2,800	—	—	—	—
Rolled Copper, ...	3,000	—	—	1,600	—
Brass, ...	2,000	—	—	—	—

TABLE 3.

Ordinary Working Stress (lb./in.^{2}).

The Straining Action producing equal stresses of opposite sign alternatively.

Material.	Tension and Compression.	Bending.	Shear.	Torsion.
Cast Iron,	1,400	2,000	1,750	—
Mild Steel,	4,300 to 5,700	4,300 to 5,700	3,300 to 4,300	2,700 to 4,000
Cast Steel,	5,700 to 7,000	5,700 to 7,000	4,300 to 5,700	4,000 to 5,300
Steel Castings,	2,700 to 4,000	3,300 to 4,700	2,300 to 4,000	2,300 to 4,000
Gun-Metal,	1,400	—	—	—

TABLE 4. (See Section IV.).

Values of a and f_c in Rankine-Gordon Formula.

MATERIAL.	f_c	VALUES OF a.		
		Case I.	Case II.	Case III.
Cast Iron,	36	$\frac{1}{6,400}$	$\frac{4}{6,400} = \frac{1}{1,600}$	$\frac{16}{9 \times 6,400} = \frac{1}{3,600}$
Mild Steel,	21	$\frac{1}{30,000}$	$\frac{4}{30,000} = \frac{1}{7,500}$	$\frac{16}{9 \times 30,000} = \frac{1}{16,875}$
Dry Timber (Strong)	3·2	$\frac{1}{3,000}$	$\frac{4}{3,000} = \frac{1}{750}$	$\frac{16}{9 \times 3,000} = \frac{1}{1,687}$

Case I.—Fixed Ends. Case II.—Hinged Ends.
Case III.—One end fixed and the other hinged.

TABLE 5—FLYWHEELS.

Average Values of q the Coeff. of Speed Fluctuation.

Usual range for Factory or Mill drives, ...	·040 to ·025
Spinning Mills—coarse to fine thread, ...	·025 to ·010
General Rough Work—punches and shears, ...	·100 to ·060
Hammering, Stamping and Crushing Machinery,	·20
Steam Engine Drives—D.C. Generators, ...	·007
Steam Engine Drives—A.C. Generators, ...	·003
Gas Engine Drives—D.C. Generators (direct), ...	·010 to ·008
,, ,, ,, D.C. ,, (belt), ...	·014 to ·0125
,, ,, ,, A.C. ,, (direct), ...	·006 to ·005
,, ,, ,, A.C. ,, (belt), ...	·008 to ·007
Large Electric Stations, ...	·0025 to ·0016
Alternators in parallel, ...	·003 to ·0003

TABLE 6.

Working Stresses on Gear Teeth at Various Speeds (lb./in.^{2}).

Material.	Pitch Circle Velocity (ft./min.).							
	0-100	200	400	600	900	1200	1800	2400
Cast Iron,	8,000	6,000	4,800	4,000	3,000	2,400	2,000	1,700
Cast Steel,	20,000	15,000	12,000	10,000	7,500	6,000	5,000	4,300
Forged Steel,	24,000	18,000	14,000	12,000	10,000	7,500	6,300	5,400
Phos. Bronze,	10,500	8,500	6,500	5,500	4,800	4,000	3,000	2,400
Raw Hide,	5,000	3,800	3,250	2,750	2,300	2,000	1,400	1,100
Nickel Chrome Steel, 100 T/in.^{2} ult.	—	—	33,000	27,500	22,000	18,500	13,500	11,000

A.E.S.D. Printed Pamphlets and Other Publications in Stock.

An up-to-date list of A.E.S.D. pamphlets in stock is obtainable on application to the Editor, *The Draughtsman*, Onslow Hall, Little Green, Richmond, Surrey.

A similar list is also published in *The Draughtsman* twice a year.

Readers are asked to consult this list before ordering pamphlets published in previous sessions.

List of A.E.S.D. Data Sheets.

1. Safe Load on Machine-Cut Spur Gears.
2. Deflection of Shafts and Beams
3. Deflection of Shafts and Beams (Instruction Sheet) — Connected
4. Steam Radiation Heating Chart.
5. Horse-Power of Leather Belts, etc.
6. Automatic Brakes (Axle Brakes)
7. Automobile Brakes (Transmission Brakes) — Connected.
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5$\frac{1}{2}$-in. diameter, subjected to Twisting and Combined Bending and Twisting.
11. Shafts, 5$\frac{3}{4}$ to 26 inch diameter, subjected to Twisting and Combined Bending and Twisting.
12. Ship Derrick Booms.
13. Spiral Springs (Diameter of Rd. or Sq. Wire).
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. ,, ,, (Plane Clutches).
17. Coil Friction for Belts, etc.
18. Internal Expanding Brakes. Self-Balancing Brake Shoes (Force Diagram)
19. Internal Expanding Brakes. Angular Proportions for Self-Balancing. — Connected.
20. Referred Mean Pressure Cut-Off, etc.
21. Particulars for Balata Belt Drives.
22. $\frac{7}{8}$″ Square Duralumin Tubes as Struts
23. 1″ ,, ,, ,, ,,
24. $\frac{3}{4}$″ Sq. Steel Tubes as Struts (30 ton yield).
25. $\frac{7}{8}$″ ,, ,, ,, (30 ton yield).
26. 1″ ,, ,, ,, (30 ton yield).
27. $\frac{3}{4}$″ ,, ,, ,, (40 ton yield).
28. $\frac{7}{8}$″ ,, ,, ,, (40 ton yield).
29. 1″ ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables)
31. Moments of Inertia of Built-up Sections (Instructions and Examples) — Connected.
32. Reinforced Concrete Slabs (Line Chart)
33. Reinforced Concrete Slabs (Instructions and Examples) — Connected.
34. Capacity and Speed Chart for Troughed Band Conveyors.
35. Screw Propeller Design (Sheet 1, Diameter Chart)
36. ,, ,, ,, (Sheet 2, Pitch Chart)
37. ,, ,, ,, (Sheet 3, Notes and Examples) — Connected.
38. Open Coil Conical Springs.
39. Close Coil Conical Springs.
40. Trajectory Described by Belt Conveyors (Revised 1949).
41. Metric Equivalents.
42. Useful Conversion Factors.
43. Torsion of Non-Circular Shafts.
44. Railway Vehicles on Curves.
46. Coned Plate Developments.
47. Solution of Triangles (Sheet 1, Right Angles),
48. Solution of Triangles (Sheet 2, Oblique Angles).
49. Relation between Length, Linear Movement and Angular Movement of Lever (Diagram and Notes).
50. ,, ,, ,, ,, ,, ,, (Chart).
51. Helix Angle and Efficiency of Screws and Worms.
52. Approximate Radius of Gyration of Various Sections.

53.	Helical Spring Graphs (Round Wire).	
54.	,, ,, ,, (Round Wire).	Connected.
55.	,, ,, ,, (Square Wire).	
56.	Relative Value of Welds to Rivets.	
58.	Graphs for Strength of Rectangular Flat Plates of Uniform Thickness.	
59.	,, Deflection ,, ,, ,, ,, ,,	
60.	Moment of Resistance of Reinforced Concrete Beams.	
61.	Deflection of Leaf Spring.	
62.	Strength of Leaf Spring.	
63.	Chart Showing Relationship of Various Hardness Tests.	
64.	Shaft Horse-Power and Proportions of Worm Gears.	
65.	Ring with Uniform Internal Load (Tangential Strain)	Connected.
66.	,, ,, ,, ,, (Tangential Stress)	
67.	Hub Pressed on to Steel Shaft. (Maximum Tangential Stress at Bore of Hub).	
68.	Hub Pressed on to Steel Shaft. (Radial Gripping Pressure between Hub and Shaft).	
69.	Rotating Disc (Steel) Tangential Strain.	Connected.
70.	,, ,, ,, Stress.	
71.	Ring with Uniform External Load, Tangential Strain.	Connected.
72.	,, ,, ,, ,, ,, Stress.	
73.	Viscosity Temperature Chart for Converting Commercial to Absolute Viscosities.	
74.	Journal Friction on Bearings.	Connected.
75.	Ring Oil Bearings.	
76.	Shearing and Bearing Values for High Tensile Structural Steel Shop Rivets, in accordance with B.S.S. No. 548/1934.	
78.	Velocity of Flow in Pipes for a Given Delivery.	Connected.
79.	Delivery of Water in Pipes for a Given Head.	
80.	(See No. 105).	
81.	Involute Toothed Gearing Chart.	
83.	Variation of Suction Lift and Temperature for Centrifugal Pumps.	
89.	Curve Relating Natural Frequency and Deflection.	
90.	Vibration Transmissibility Curved for Elastic Suspension	Connected.
91.	Instructions and Examples in the Use of Data Sheets, Nos. 89 and 90.	
92.	Pressure on Sides of Bunker.	
93-4-5-6-7.	Rolled Steel Sections.	
98-9-100.	Boiler Safety Valves.	
102.	Pressure Required for Blanking and Piercing.	
103.	Punch and Die Clearances for Blanking and Piercing.	
104.	Nomograph for Valley Angles of Hoppers and Chutes.	
105.	Permissible Working Stresses in Mild Steel Struts with B.S. 449, 1948.	
106.	Compound Cylinder (Similar Material) Radial Pressure of Common Diameter (D1).	

(Data Sheets are 3d to Members, 6d to others, post free).

Orders for Pamphlets and Data Sheets to be sent to the Editor, *The Draughtsman*, cheques and orders being crossed "A.E.S.D."